THE UNIVERSITY OF ALBERTA

A COMPARISON OF PROGRAMMED AND CONVENTIONAL MATHEMATICS ENRICHMENT MATERIALS OVER TWO GRADE SEVEN MATHEMATICS ACHIEVEMENT LEVELS

by

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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommended to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Comparison of Programmed and Conventional Mathematics Enrichment Materials over Two Grade Seven Mathematics Achievement Levels" submitted by David Joseph Bale in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of the study was to test a mathematics enrichment program which would be practical for the small junior high school and at the same time provide differentiated enrichment for different types of potentially talented students.

The differentiated enrichment consisted of two supplementary texts on the topic of finite mathematical systems, one programmed and the other conventional. The two different types of potentially talented mathematics students consisted of those seventh graders with high general ability and high mathematics achievement, and those with high general ability but mathematics achievement not commensurate with their ability. The sample consisted of 120 students, 60 of each type.

During the spring of 1966 both categories of students studied the enrichment materials on their own for one 30-minute period each week for eight weeks. There were two identical experiments, one for each category of students. For each experiment, one treatment group studied the conventional materials, another studied the programmed materials, and a third group, the control, continued with regular mathematics classwork. Post tests of regular mathematics classwork achievement were used as criteria in the one-way analyses of covariance which were carried out on the data to test the null hypotheses. Previous mathematics achievement, I.Q., and mathematics attitude were the covariates.

For both categories of students it was found that there was no significant difference in regular class mathematics achievement among the three groups. In enrichment mathematics, there was no significant difference among the three groups of high ability students with mathematics achievement not commensurate with their ability. However, for those with high mathematics achievement as well as high general ability, the enrichment test scores of the

conventional group were significantly higher than those of the programmed and control group. When the analysis was performed on both categories combined, it was found that there was no significant difference between the conventional and programmed group scores on the regular and the enrichment mathematics achievement test scores. However, the enrichment test scores of each of these groups were significantly higher than those of the control group.

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CHAPTER I

THE NATURE AND SIGNIFICANCE OF THE PROBLEM

I. INTRODUCTION

Whatever a student's capabilities may be, he should be given the opportunity and encouragement to develop his talents. The importance of this philosophy is expressed by Vance in the following way:

> We are probably unaware of the real brilliance of the occasional genius and have difficulty in recognizing it. History records many instances in which the potential of a mathematical genius went unnoticed. In these instances the potentials of the individuals were developed in spite of the school. We cannot surmise a guess as to the number of individuals with equally great potential who did not develop.[1]

Enrichment is one well recognized method of providing students with the opportunity of developing their talents. We must be able to design suitable enrichment programs so that all potentially talented students will be able to benefit from them.

II. BACKGROUND TO THE PROBLEM

In September 1965 new mathematics courses[2] were introduced into grade seven in Alberta. The Edmonton Separate School Board were concerned with incorporating a successful enrichment program with the new curriculum. They showed their concern by supporting this study.

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[1]E.P. Vance (editor), Program Provisions for the Mathematically Gifted Student in the Secondary School, (Washington, D.C. National Council of Teachers of Mathematics, 1957), p. 20.

[2]Henry Van Engen, Maurice L. Hartung, Harold C. Trimble, Emil J. Berger, Ray W. Cleveland, and A.B. Evenson, Seeing Through Mathematics, Book I, (Toronto: W.J. Gage Limited, 1964); and Mervin L. Keedy, Richard E. Jameson, and Patricia L. Johnson, Exploring Modern Mathematics, (New York: Holt, Rinehart and Winston, Inc., 1963).

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At the outset several problems were recognized. First, in many enrichment programs the students were selected on the basis of a set of criteria which had been determined experimentally. Those students who did not measure up to the criteria were eliminated from the program. The problem with this procedure is that, in the experimental determination of the selection criteria, usually only one enrichment program was used. It may be that the reason for the failure of some students in the experiment was the unsuitability of the program. Perhaps a suitable enrichment program would ensure their success.

Usually when the suitability of different kinds of enrichment programs for potentially talented students has been tested, the students selected have been easily identifiable as superior. It may be that some students who were not so easily identifiable as superior might have been successful in some of the programs.

The conclusion to be drawn from the foregoing discussion is that the procedures used in selection of students and programs for enrichment have not ensured that all potentially talented students have had the opportunity to receive suitable mathematics enrichment experiences.

A second problem identified was that most enrichment practices described in the literature required resources not usually available in the typically small junior high schools of the Edmonton Separate School System. Concern for providing a suitable mathematics program for all students has created within the system interest in instructional grouping within classes and provision of mathematics enrichment during regular class periods. Consequently, a mathematics enrichment program designed to test solutions to these problems and concerns became the subject of this investigation.

III. THE PROBLEM

Statement of the Problem

The areas of concern which emerged in the foregoing discussion led to the formulation of the following two problems:

1. To determine if potentially talented mathematics students can benefit from mathematics enrichment work on their own during regular class periods without loss in regular mathematics classwork achievement.
2. To compare the effectiveness of programmed and conventional enrichment materials for each of two categories of potentially talented mathematics students classified on the basis of level of previous mathematics achievement.

Definitions

Potentially Talented Mathematics Students - those students who adhere to Hlavaty's description of mathematically talented students. This description is given in Chapter III.[3]

Level I Achievers - seventh grade mathematics students with I.Q.'s of 115 or more whose scores were among the top 60 on the Mathematics Achievement Pre-Test.

Level II Achievers - seventh grade mathematics students with I.Q.'s of 115 or more whose scores were among the bottom 60 on the Mathematics Achievement Pre-Test.

Programmed Enrichment Material - the text Modular Systems by Edward J. Zoll, described in detail in chapter III, and used by the programmed treatment group.

Conventional Enrichment Material - the text Finite Mathematical Systems by M. Scott Norton, described in detail in chapter III, and used by the conventional treatment group.

Hypotheses

Central Question A: Does the practice of allowing students to be absent from mathematics class for one period each week for eight weeks to engage in mathematics enrichment work result in loss in achievement in regular mathematics classwork?

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[3] *Supra.*, p. 13 - 14.

Null Hypothesis 1: With seventh grade students, there is no significant difference at the five percent level in regular mathematics classwork achievement among the control and experimental groups within

a) Level I,

b) Level II,

c) Levels I and II combined.

Central Question B: Which will lead to better achievement in enrichment mathematics, programmed or conventional materials?

Null Hypothesis 2: With seventh grade students, there is no significant difference at the five percent level in enrichment mathematics achievement among control and experimental groups within

a) Level I,

b) Level II,

c) Levels I and II combined.

Delimitations

The experiment was limited to the population of seventh grade mathematics students of the Edmonton Separate Schools with I.Q. scores of 115 or more, and the two types of treatment as defined.

Achievement in regular mathematics classwork and enrichment mathematics work, the attitude of students toward mathematics, and the attitude of teachers toward the experiment were the only effects of the experiment to be determined.

IV. SIGNIFICANCE OF THE PROBLEM

Little has been done to solve the problem of enrichment in the small junior high school. The literature is overflowing with suggestions such as library research, projects, films, seminars, field trips, and committee work, but the small junior high school has neither the staff nor the material resources to adopt such programs. Independent study of a limited amount of suitable mathematics enrichment material might prove to be more practical for this kind of school.

Since the success of students in this kind of enrichment program depends a great deal on their ability to work independently, a comparison of their success using conventional versus programmed enrichment materials would seem appropriate. The literature supports the conjecture that the programmed materials should prove to be superior, but there is a need to test this under the special procedures used in this particular experiment.

The importance of this study is as follows: it would be an attempt to devise a mathematics enrichment program that is practical for the small junior high school; it would test the suitability of the two different kinds of enrichment materials for different kinds of potentially talented students.

V. EXPERIMENTAL SETTING

The Edmonton Separate School Board granted permission for the experiment to be undertaken in their schools during the spring of 1966. The sample consisted of 120 seventh graders with an I.Q. of 115 or more. About two-thirds (355 students) of the total population from which the sample was derived wrote a mathematics achievement pre-test based on the mathematics classwork taken up to February of that school year. Those whose scores were among the top 60 and those whose scores were among the bottom 60 on this test were selected and classified as Level I and Level II respectively.

For each level separately, 20 students were assigned randomly to each of the two treatment groups and the control group. The two experimental treatment groups studied their respective enrichment materials independently for one 30-minute period each week for a duration of eight weeks while the control group continued to work on regular mathematics at these times. At the conclusion of this period, all the subjects were given a mathematics attitude questionnaire, and post tests in regular classwork and enrichment mathematics. Also at this time, a questionnaire on the program was administered to the teachers.

The data collected were subjected to one-way analyses of covariance,

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using the two mathematics post tests as criteria and the mathematics pre-test, attitude, and I.Q. as covariates.

The findings of this report are based on this statistical analysis of the data.

VI. OUTLINE OF THE REPORT

The present chapter is an introduction to the study. Chapter II consists of a review of the literature that is related to the study. A detailed description of the design of the experiment and the statistical procedures used in analyzing the data is presented in chapter III.

In chapter IV the results from the statistical analyses and the teacher questionnaire are presented. A summary of the investigation together with the conclusions, limitations, and implication for further research are presented in chapter V.

CHAPTER II

THE REVIEW OF THE LITERATURE

This study is primarily concerned with testing the suitability of different kinds of mathematics enrichment materials for different kinds of potentially talented students during regular class periods. Therefore, the aspects of the literature reviewed here will be as follows: the identification of the gifted student, and more particularly the identification of the mathematically talented student; a brief description of the programs of provision for the gifted student, including enrichment materials, enrichment plans and their uses; mention of research on factors affecting achievement in mathematics; experimental studies in mathematics enrichment; experimental studies in programmed learning.

I. IDENTIFICATION OF THE MATHEMATICALLY TALENTED STUDENT

One of the major problems of creating special educational opportunities for potentially talented students is that of identifying them. There is much confusion and disagreement over selection criteria and terms used to identify these kinds of students. When attempts have been made to determine selection criteria experimentally, a single program has been used as a criterion of success. In this case, the reason for failure of some of the students may be the unsuitability of the particular enrichment program for them rather than inadequacy on their own part. Instead of using information concerning students' educational characteristics for identifying and selecting one category of obviously superior students, we could use this information to select different categories of potentially talented mathematics students and attempt to determine a suitable enrichment program for each category.

The Meaning of the Term Gifted

According to Monroe, "the term 'gifted children' has traditionally been applied to those of superior intelligence as measured by standard

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psychological tests."[1] At first, little attempt was made to identify specific giftedness, and most of the research on characteristics of the gifted child has consisted of studying children selected on the basis of general intelligence. For instance, in the classic study of the gifted child carried out by Terman during the twenties and thirties, one thousand subjects with an I.Q. of 140 or more were selected.[2] DeHaan classified one tenth of one percent of the population as gifted of the 'first order', whereas the top ten percent of the population were the gifted of the 'second order'. For the top ten percent the cut-off point is an I.Q. of 120.[3] Hollingsworth selected 130 as the cut-off point of I.Q. of the students used in her studies of gifted children, but Bentley went as low as an I.Q. of 110 for the same purpose.[4] Hence, there seems to be considerable difference of opinion as to whom the term gifted should apply to in relation to general intelligence.

Furthermore, there is some confusion over the terms used to describe high ability students. They are variously described as gifted, exceptional, talented, superior, bright, able, creative, and rapid learners. Sometimes the terms are meant to be synonymous, but at other times it is difficult to determine if they mean the same thing. Part of the difficulty is that the characteristics of the gifted are by no means clear or certain. For instance, some students possess high ability in specific areas such as music, art, or mathematics, but not necessarily high general ability. Furthermore, some students display what is thought to be creative qualities without

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[1] Paul Monroe, "The Gifted Child", Encyclopedea of Educational Research, (3rd ed.), p. 584.

[2] Ibid.

[3] Robert F. DeHaan and Robert J. Havighurst, Educating Gifted Children, Revised and Enlarged Edition (Chigaco: The University of Chicago Press, 1961), p. 45.

[4] Monroe, Loc. cit.

necessarily exhibiting high intelligence or achievement. Because of this lack of specific definitions of any of the terms mentioned, students displaying high ability in one area or another are often lumped together under one or more of the terms. To illustrate further the point that the characteristics of the gifted may vary substantially, Scheifele's comprehensive list of the intellectual characteristics of the gifted indicated that they show as much unevenness in abilities in the subject matter areas as other children.[5] Thus, although students of exceptionally high general intelligence can be considered as the gifted, this is not a sufficiently satisfactory definition if we are concerned with students of high ability in a specific area such as mathematics.

Some observers even disagree with the use of intelligence as a criterion at any time. According to Monroe, "some writers such as Ernst, indicate their distrust of intelligence tests, and in the writings of some educators who are not primarily psychologists there appears an undersirable tendency to identify giftedness with achievement and good citizenship."[6] Even Terman and Oden noticed a tendency for I.Q.'s of very bright children to decrease with age. They suggested that this may be due in part to the unreliability of testing procedures. Possibly the narrowness of the curriculum accompanied by a lack of direct provision for the gifted may be partly responsible. Fitzgerald has suggested that narrow curriculum is responsible for the lower variability in arithmetic achievement than in reading achievement.[7] Hence, since general intelligence is undoubtedly

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[5]Marian Scheifele, The Gifted Child in the Regular Classroom, (New York: Bureau of Publications, Teachers College, 1953), p. 6.

[6]Monroe, Loc. cit.

[7]William Morley Fitzgerald, A Study of the Factors Related to the Learning of Mathematics by Children in Grades Five, Seven, and Nine, Unpublished Doctroral Dissertation (University of Michigan, 1962), p. 22.

related to environmental influences, it is quite conceivable that the deprivation caused by an overall narrow curriculum could contribute to the lowering of I.Q. Thus, while I.Q. as a criterion of giftedness seems to be useful, "there is much agreement that the intelligence test should be supplemented by other evidence..."[8]

Identification Through Observation of Characteristics

Scheifele's and the Secondary School Curriculum committee's lists of the intellectual characteristics of the gifted student are probably the most complete. Scheifele says that:

Intellectually, the gifted child in relation to other children, tends to:

A. Possess superior ability in reasoning, generalizing, dealing with abstractions, comprehending meanings, thinking logically, and recognizing relationships.
B. Perform highly difficult mental tasks, an ability described as 'power.'
C. Learn more rapidly and easily
D. Show intellectual curiosity.
E. Possess superior insight into problems.
F. Have a wider range of interests.
G. Show greatest superiority in reading ability, both in speed and in comprehension; language usage; arithmetic reasoning; science; literature; and the arts.
H. Do effective work independently.
I. Apply orginality and initiative in intellectual tasks.
J. Show less patience with routine procedures and drill.
K. Exhibit alertness, keen observational ability, and quick response.
L. Show as much unevenness in abilities in the subject matter areas as other children.
M. Have a longer interest span; show more interest in abstract than in practical subjects; exhibit greater superiority in attainment in abstract subjects and less in manual activities.
N. Have an interest in the future, a concern with origin, destiny, and death though emotionally unable to accept realities of the latter.[9]

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[8]Monroe, Loc. cit.

[9]Marion Scheifele, The Gifted Child in the Regular Classroom, (New York: Bureua of Publications, Teachers College, 1953), p.6.

The Secondary School Curriculum Committee of the National Council of Teachers of Mathematics lists the following as the general and special characteristics of the gifted in mathematics:

A. Recognizes patterns readily and enjoys speculating on generalizations.
B. Prefers to think on higher levels of abstraction.
C. Classifies particular cases as special cases of more general situations with relative ease.
D. Follows a long chain of reasoning, frequently anticipating and contributing.
E. Frequently asks profound questions.
F. May be reading mathematics books years ahead of his class.
G. Is frequently impatient with drill and details that he thinks are not important.[10]

Fehr considers that there is a lack of adequate means of identifying giftedness as described in such lists other than through formal tests.[11] However, Johnson states that as far as he knows there are no published tests that will identify students having the characteristics which he lists and that observation is the method that must be used.[12]

In identifying the gifted student through observation there can be considerable error. In a study by Lewis, described by DeHaan, teachers using their own interpretation of the criterion of genius selected 341 children from a large sample. The sample was tested with various standardized tests. Forty percent of the 341 had I.Q.'s of 120 and above, and the other sixty percent had I.Q.'s ranging from 119 to below 70.[13] Fehr discussed certain studies in which only fifteen out of one hundred were correctly selected by observation as gifted.[14] Hence, it is apparent that selecting students by observation alone is not a satisfactory method.

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[10] The Secondary Mathematics Curriculum, Report of the Secondary School Committee (Washington, D.C: National Council of Teachers of Mathematics, May 1959), p. 410.

[11] Howard F. Fehr, "General Ways to Identify Students with Scientific and Mathematical Potential," The Mathematics Teacher, XLVI (April, 1953), p. 230.

[12] Donovan Johnson, "Enriching Mathematics Instruction with Creative Activities," The Mathematics Teacher, XLVI (May, 1953), p. 322.

[13] DeHaan, Op. cit., pp. 166-167. [14] Fehr, Loc. cit.

Identification Through Testing

The fallacy of identifying the gifted by means of using general intelligence as the only criterion has already been discussed in some detail. There seems to be no studies which isolate one or two other types of tests such as aptitude or standarized achievement tests, although some work has been done on combining the results of these types of tests with tests of general intelligence for the purpose of identifying the gifted. Vance says that

> Scores on aptitude and achievement tests are significant, but not enough alone. Grades in previous mathematics courses should be considered, but they can be deceiving. They may represent the extent to which the student has conformed to the teacher's thinking.[15]

Some studies have been attempted which purport to identify the factors related to the achievement of all students in a specific subject. In the area of mathematics, one such study was conducted by Fitzgerald. He concluded that arithmetic age and educational age (the average of arithmetic, language, and reading ages) of a cross-section of students in grades five, seven, and nine correlated most highly with achievement in two units of special modern mathematics material. The material was, in effect, enrichment material, although it was introduced for reasons other than enrichment, and all students in the classes participated in the experiment. Hence, there was no attempt made to ascertain the correlations of the seventeen factors considered in the experiment with the achievement of gifted students only.[16] The consensus of opinion seems to be that the identification of the gifted through testing is merely one facet of a broader procedure.

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[15]E.P. Vance (editor), Program Provisions for the Mathematically Gifted Student in the Secondary School (Washington, D.C: National Council of Teachers of Mathematics, 1957), p. 10.

[16]Fitzgerald, Loc. cit.

Identification by Means of Combining Several Criteria

If, as seems to be indicated, gifted students must be identified on the basis of a combination of several criteria, a systematic system of selection of criteria and suitable weighting procedures must be adopted. Hlavaty stated this in the following way:

> All available pertinent data including information from intelligence and achievement tests and from interest inventories; information on previous school performance; staff evaluations, anecdotal records, etc; other types of data from cumulative record...Flexibility in the application of any set of criteria is necessary. It must be recognized that errors are possible in both the assembling and interpreting of data. At certain decision points, some of the criteria should be accorded greater weight than others.[17]

There is a lack of experimental and other studies in this area of combining several criteria and weighting them suitably in an attempt to identify the gifted student.

At first glance, a possible further complication seems to be that there are different varieties of talent in subjects such as mathematics. Hlavaty suggested the following categorization of potentially talented mathematics students:

> Group A consists of students readily identifiable because of high general intelligence, outstanding accomplishment in all courses throughout their school careers, and deep interest in and satisfaction derived from the study of mathematics. Students in this category are usually highly motivated to continue with all the mathematics available to them at the secondary level.
>
> Group B consists of students of high measurable ability, but whose performance in mathematics has not been commensurate with their ability. possible reasons for their underachievement are: inadequate motivation; a negative attitude--sometimes due to poor instructional procedures; lack of insight into the areas of learning for which mathematics is required.
>
> Group C consists of students from low socio-economic areas in which environmental influences in the community, in the home, and even in the school have served to deprive students of the opportunities to develop innate abilities to the extent that they can be measured accurately by our present evaluation instruments. Here it should be recognized that intelligence tests reveal to a large degree only "developed ability", not innate intelligence.

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[17]Julius H. Hlavaty (editor), Mathematics for the Academically Talented Student, A Report, National Education Association Project on the Academically Talented Student and the National Council of Teachers of Mathematics, (Washinton, D.C: National Council of Teachers of Mathematics, 1959), p. 10.

> Finally, there is group D, students of high general intelligence, whose general ability in their high school course is mediocre, but who, because of motivation and special talents, have achieved outstandingly in mathematics.[18]

Hlavaty's classification was an attempt to identify the basic categories into which potentially talented mathematics students fall. If we assume that this categorization is useful, the appropriate combination of criteria of selection will be suggested by the description of the students in each category. Furthermore, the problem of suitable weighting will be eliminated or greatly reduced. In addition, classification of mathematically talented students may be the key to adequate provision for them, for it may be that different methods for different categories would be appropriate. Hlavaty suggested that the main criteria that should be used in selecting and classifying mathematically talented grade seven and eight students should be: an I.Q. of 115 or more; scores on standardized achievement tests two or more years above the grade in reading comprehension, and one or more years above the grade in arithmetic comprehension, concepts, and problem solving; above average school marks; and an interest in mathematics.[19] Of course, different weighting of each of these criteria would be used for different cate gories of students. For instance, little weight would be given to mathematics achievement for category B students.

<u>Summary</u>

The literature on the gifted child points out sharply that attempts to identify such students on the basis of one or two criteria only are futile. There is general agreement that a wide variety of data must be used. However, this presents the problem of interpretation of the data. This seems even more complicated because there are varieties of potential mathematics talent. If the various kinds of potentially mathematically talented students can be classified into a few basic categories, the problems of selection of students and proper provision for them would be markedly reduced.

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[18] *Ibid.*, pp. 10-11.

[19] *Ibid.*, p. 12.

II. PROVISION FOR THE GIFTED CHILD

The three most common practices used in providing for the gifted child are ability grouping, acceleration, and enrichment. Ability grouping, rarely used in most parts of Canada, seems to have fallen into disfavour in the United States during the last decade mainly for the following reasons: the special kinds of teachers required are scarce; enrichment is still required for some students because there is still a variety of abilities exhibited; the smaller schools find it less feasible. Acceleration has also been criticized recently. However, limited acceleration accompanied by an enriched curriculum is still looked upon with some favour. Hence, enrichment, even if it is being used to complement acceleration or ability grouping, appears to be the most important way of providing for talented students. Providing enrichment for students within the class rather than outside seems to be more popular when more than a few students in a class are to be considered.

Acceleration, Ability Grouping, and Special Programs

Monroe has said that ability grouping and acceleration are administrative rather than teaching procedures. He feels that much has been written about the harmful social effects of acceleration, but that the research shows that these exist only in cases where students have been accelerated on the basis of unsound identification procedures.[20] When Monroe speaks of ability grouping, he does not mean groupings within classes. Within class grouping and enrichment are complementary to each other and will not be discussed in this section. The type of ability grouping which is discussed here is often referred to as homogeneous grouping or streaming. Hall feels that many of those opposed to homogeneous grouping are so disposed because they feel that it is impossible to accomplish. The variations of ability and achievement in these classes will still be quite wide and will likely increase if the students are suitably challenged. Moreover, if the grouping is not

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[20] Monroe, Loc. Cit.

by subject, there is likely to be a wide variation of the abilities of the students with respect to the different subjects. In describing the practice of ability grouping, Hall states that while it is quite common in the United States and has been in existence in parts of Europe for a great many years, it is not common in most parts of Canada, especially Alberta.[21]

Since ability grouping and acceleration are administrative procedures, they would likely be used only under special circumstances. For instance, homogeneous grouping would be more practical in large schools with special facilities and staff. These procedures are impractical for small schools with limited facilties. Furthermore, even though these administrative procedures may be used extra enrichment or special more advanced enriched courses are an integral part of them. Special programs may consist of such things as multi-track programs, special honors classes, after school or summer classes, mathematics clubs, and so on. Multi-track programs have been attacked by Van Engen. He feels that a one track program consisting of good mathematics taught so as to take into consideration the rate at which children learn mathematics is better.[22] Although special programs are being carried on in a number of places in the United States, evaluations of these have been largely subjective. Very little in the way of special mathematics programs exists in Canada.

After discussing the problems of these methods, Dyer, Kalin, and Lord stated that enrichment is an important aspect of within-class grouping.

> A closely related problem is that of getting enough flexibility into the arithmetic curriculum to serve the bright, the not-so-bright, and the dull equally well. Most school authorities probably recognize that some pupils

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[21]Lynn G. Hall, "A Review of Enrichment Procedures for Gifted Pupils," Alberta Journal of Educational Research, IV (June, 1958), p. 117.

[22]H. Van Engen, "Multi-track Programs! "The Mathematics Teacher, LLII (March, 1959) pp. 205-206.

> can learn arithmetic much faster than others, but recognition of the phenomenon is a long way from knowing how to cope with it in actual classroom practice. Some schools have tried grouping students according to ability in one way or another, but for various reasons have found it expedient to abandon this device. Opinion now favours "enrichment" for the abler pupils--giving them something interesting to do while the slower pupils are being brought up to par in the regular subject matter.[23]

Enrichment

Several important decisions face the prospective enrichment program organizer. He must decide whether the program is to take place within or outside the regular class, what activities the students are to engage in, what content and materials are to be used, and what kind of presentation of content would be best. A number of factors, particularly the size of the school, may have a bearing on these decisions.

> The kinds of modifications needed in a particular school for the development of more adequate mathematics experiences for gifted students will be determined by the size of the school, nature of the professional staff, community setting, and similar factors.[24]

Another factor which will have a bearing on these decisions is the nature of the students for whom the enrichment is designed. With regard to within versus outside the class programs, Harris said:

> The precocious eight-year-old who attacks complex issues in physics and mathematics truly does present a problem to the average classroom teacher. That teacher is likely to want him to have special opportunities outside her classroom. If we think of all children with I.Q.'s over 115 or 120, and the variety of special aptitudes present, we inevitably think of the many individual differences within this group and are concerned with how the group blends in with the more typical selection of children. We are, therefore, less concerned with the usual educational methods and are more likely to speak of enrichment within the regular classroom.[25]

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[23] Henry S. Dyer, Robert Kalin, Frederick Lord, Problems in Mathematics Education, (Princeton, N.J., Educational Testing Service, 1956), p. 19.

[24] New Developments in Secondary School Mathematics, "Mathematics and the Gifted Students-Some problem Areas", (Washington, D.C., National Council of Teachers of Mathematics, 1959), p. 70.

[25] E. Paul Torrance, Editor, Talent and Education, Papers presented at the 1958 Institute on Gifted Children, (Minneapolis, University of Minnesota Press, 1960), p. 127.

Lewis supported within-class enrichment but pointed out that it is an unproven method.

> There are many supporters of enrichment within the regular classroom as a means of challenging talented children. Unfortunately, however, there is very little research evidence bearing on the effectiveness of this method.[26]

In support of the success of this practice, he cites a 1924 study of the Winnetka plan by Washburn and a 1954 study in the Cedar Rapids Schools as two of the few reported studies.

Monroe said, "enrichment programs in general tend to cover all subjects, often with special procedures designed to stimulate intellectual curiosity, minimize repetitious activites not needed for bright children, and permit more freedom for exploration by students."[27] He goes on to say that seminar procedures, group discussions, research or investigative activities, studies of an evaluative sort, and the like are often included.

Johnson felt that differentiation in assignments was probably the simplest and often the most practical method of providing for superior students.[28] Jackson described a much more extensive program of student activities as carried out in Minneapolis junior and senior schools in the late fifties. The program for superior students in the regular classroom would include many of the following activities:

> Doing library research work, reading for interest or recreation, writing book reports, giving oral reports, solving additional and more difficult problems, investigating topics of special interest, making models, preparing exhibits for science fairs, making scrapbooks, watching and reporting on mathematics television shows, organizing and conducting meetings of a mathematics club.[29]

However, with regard to small schools, Lead said:

> With heterogeneous groups, which will invariably be the case in smaller schools, the teacher must vary his teaching techniques and adjust the work within the class to suit the varying abilities of its members (for instance, by giving two or more types of assignments in

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[26] _Ibid_., p. 83.

[27] Monroe, _Op. cit_., p. 12.

[28] Johnson, _Loc. cit_.,

[29] Harvey O. Jackson, "The Superior Pupil in Mathematics", _The Mathematics Teacher_, LLI (March, 1959), p. 203.

by Hlavaty for grades seven and eight consists of studying the properties of other numeration systems in order to better understand the base ten system, properties of number systems, intuitive geometry, and the beginnings of deductive reasoning.[34] These topics are now included in the regular courses for these grades in Alberta and in many other places throughout the continent.

Several things are apparent from the discussion so far. First, enrichment implies that access to special materials is necessary. Second, assuming that a combination of horizontal and vertical enrichment is used, the procedures vary mainly in the amount of involvement in a variety of activities on the part of the student. If any of these activities involve independent work, it may be wasteful without much guidance or the use of programmed material as a substitute for some of this guidance. Finally, many of these activities will be limited by the size of the school and availability of suitable materials.

According to Monroe,

> Most reports indicate that students profit markedly from their experiences in enrichment programs. The evaluation criteria tend to be general, and there is an absence of clear-cut, convincing evidence. The findings tend to be based on stated opinions of teachers and administrators, parents and students; it is clear that opinions are generally favorable to the special programs.[35]

Experimental Studies of the Effects of Enrichment

Most of the experimental studies of enrichment deal with comparing the effects of enriched and non-enriched programs on the achievement and attitude of superior students.

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[34] Hlavaty, Op. cit., pp. 23 - 24.

[35] Monroe, Op. cit., p. 588.

the same topic, one more difficult than the other). (Other techniques are elastic assignments, directing superior students to do independent work in the library, special projects or work in supplementary texts while remedial classes are held for the slower learner.)[30]

Dyer felt that the scarcity of suitable published enrichment materials and the difficulty of meshing them smoothly with the rest of the curriculum have contributed to the failure of many mathematics enrichment programs.[31] Furthermore, Dockrell cites research that shows that teachers are not very successful in designing their own enrichment materials.[32] Another criticism of some enrichment programs has been made by Ernst. While Witty and Lehman feel that the bright student does much to help himself, Ernst feels that this process of self help in many cases is wasteful, and that a need for adult guidance in some cases is felt.[33]

Programmed learning with its characteristics of small sequential steps, reinforcement, and branching might prove to eliminate this problem. Few experiments have been concerned with comparing the effectiveness of programmed and conventional text presentations for enrichment purposes.

In discussing enrichment content, a distinction is usually made between two types. Vertical enrichment or enrichment in depth is accomplished by providing for specialization or more mature work in a given field. Horizontal enrichment or enrichment in breadth gives a student opportunities to work in areas not normally included in the child's school experience. A combination of these two types is generally considered desirable.

Since the introduction of modern mathematics into the curriculum, much of the material suggested for junior high school enrichment programs is now part of the regular course content. For instance, the material recommended

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[30]New Thinking in School Mathematics, Report of a Seminar Held by the Canadian Teachers' Federation at Ottawa, April 28-30, 1960, (Ottawa: Canadian Teachers' Federation, 1960), p. 143.

[31]Dyer, Loc. cit.

[32]W.B. Dockrell, Streaming, (unpublished paper, University of Alberta, 1964), p. 4 (Mimeographed).

[33]Monroe, Loc. cit.

Ray[36] conducted a longitudinal study of the effects of enriched and accelerated programs on the attitude and achievement of superior junior high school students. He concluded that both the accelerated and the enriched programs were better than the regular program, that the students felt there was much value in written and oral reports, and that the students felt the projects and films did not help them in the way that they understood they were intended to.

Several studies have been concerned primarily with the achievement aspects of enrichment. Ten first year algebra classes of four teachers were divided into experimental and control groups in an experiment conducted by Albers and Seagoe.[37] Only students with an I.Q. of 120 or more were used in the study. The two groups were matched on initial algebra achievement, chronological age, and mean I.Q. The experimental group, who were informed that they had been selected for special study because they were capable, used fifteen percent of their normal algebra class time plus certain additional alloted time for their study or discussion of enrichment topics following the outlines provided for the purpose of broadening and deepening the knowledge of the content of the regular course. The results of the experiment were that there was no loss in achievement in the regular work of the course and significantly greater achievement on a test of knowledge of enrichment materials for the experimental group.

A study of ninety-eight grade eleven students to see if an enrichment program in advanced algebra brought significant gains for talented students

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[36]John James Ray, A Longitudinal Study of the Effects of Enriched and Accelerated Programs on Attitude Toward and Achievement in Eighth Grade Mathematics and Ninth Grade Algebra, Unpublished Doctoral Dissertation, (Indiana University, 1961).

[37]Mary E. Albers and May V. Seagoe, "Enrichment for Superior Students in Algebra Classes," Journal of Educational Research, (March, 1947), pp. 481-495.

[illegible]

[illegible]

[illegible]

[illegible]

in heterogeneous classrooms was conducted by Long.[38] He also sought to determine whether enrichment in the regular classroom affected the achievement of the non-talented and to see if the program produced any attitude changes. Group A, the experimental group, consisted of fifty-four students of whom fifteen were classified as talented, whereas group B, the control, contained forty-four students of whom eleven were talented. Group A participated in enrichment activities such as supplemental reading and reports, special projects, serving as group leaders, working review lists for extra credit, participating in mathematics contests, and giving discussions on new materials and topics. The results showed that the talented of both the control and the experimental groups gained significantly more in achievement than the whole of both groups and that the group A talented students achieved greater gains than the group B talented. Furthermore, students reactions of the experimental group were slightly more favorable toward the study of mathematics than were those of the control group. This study is significant in that students of a wide range in ability were subjected to the enrichment and the results favored the talented students.

Wilson[39] studied the longitudinal effects of shortened ninth grade algebra periods replaced by seminar periods in which enrichment materials and procedures were used. The study took place over a period of three years, and the result was that there was a significant difference in achievement in favor of the experimental group.

Several studies have been concerned with enrichment for talented students by means of accelerating their mathematics programs. Passow and Goldberg[40]

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[38]Roy Gilbert Long, A Comparative Study of the Effects of an Enriched Program for the Talented in Advanced Algebra Classes, Unpublished Doctoral Dissertation (Indiana University, 1957).

[39]J.A.R.Wilson "Some Results of an Enrichment Program for Gifted Ninth Graders", Journal of Educational Research, LIII (December, 1959), pp. 157-160.

[40]A. Harry Passow, Mirima L. Goldberg, and Frances R. Link, "Enriched Mathematics for Gifted Junior High School Students", Educational Leadership, XVIII (April, 1961) pp. 442-448.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Hartung,[41] and Justman,[42] all conducted such studies at the junior high level with the identical result that the achievement of the accelerated students was significantly better than that of the non-accelerated groups.

Some studies have attempted to determine the effects of enrichment programs on the attitude of the students toward mathematics, but most of them have consisted of surveys of the opinions of teachers and administrators. Some of the studies already mentioned reported that there was a significantly better improvement in attitude of students subjected to enriched programs. In the Albers and Seagoe[43] study the results of an interest inventory verified this fact. Passow, Goldberg, and Link[44] also confirmed this result on the basis of an attitude test.

Payne[45] reported a more enthusiastic attitude on the part of students following individual study of enrichment units which he prepared. He attributed this to the motivation created by the newness and novelty of the materials.

Summary

It seems that mathematically talented students should study enrichment materials. It is also apparent from the research that when they do there is a significant gain in achievement and attitude. However, there

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[41] Maurice L. Hartung, "High School Algebra for Bright Students," The Mathematics Teacher, XLVI (May, 1953), pp. 316-321; 325.

[42] Joseph Justman, "Academic Achievement of Intellectually Gifted Accelerants and Non-accelerants in Junior High School," School Review, LXII (March, 1954), pp. 142-150.

[43] Albers and Seagoe, Op. cit., p. 490.

[44] Passow, Goldberg, and Link, Loc. cit.

[45] Joseph N. Payne, "Self-instructive Enrichment Topics for Bright Pupils in High School Algebra", The Mathematics Teacher, LI (February, 1958), pp. 115-116.

seems to be a lack of experimental studies comparing different enrichment procedures. Furthermore, most of the studies used easily identifiable superior students, with the result that we have little information about the many different types of mathematically talented students who are not so readily identified, and the effects of enriching the mathematics programs of these students.

III. PROGRAMMED LEARNING

Much experimentation with programmed learning has taken place during the sixties. Schramm summarized a great deal of this research and found that eighty percent of it dealt with testing and comparing various kinds or parts of programs. He also found that

> Among the remaining research experiments there are a considerable number of evaluative tests which seek to compare the amount of learning from programs with conventional classroom teaching of the same subject. A few experiments with special applications of programs - to slow learners, deaf children, to industrial trainees, to voluntary and individual users, and so forth. A few others are concerned with the use of programs for special objectives, such as discovery teaching or transfer of training.[46]

There is practically no research on comparing programmed and conventional texts for enrichment. There is evidence however, that programmed learning is being used in the classroom for enrichment nearly as much as for any other single purpose.[47]

In answer to the question, do students learn from programmed instruction, Schramm said,

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[46] Wilbur Lang Schramm, The Research on Programmed Instruction, an Annotated Bibliography, U.S. Office of Education, (Bulletin No. 35, Washinton, D.C: U.S. Department of Health, Education, and Welfare, 1964) pp. 2 - 3 .

[47] A Survey of the Use of Programmed Instruction in Canadian Schools, 1962 - 1963, (Research Memo No. 12; Ottawa: Research Division, Canadian Teachers Federation, September 1963).

> They do indeed learn...But the question, how well they learn from other kinds of instruction, we cannot answer quite confidently...let us record some of the evidence comparing programs with conventional classroom instruction. We have tabulated 36 such reports. ...Of the 36 comparisons, 18 showed no significant difference when the two groups were measured on the same criterion test. But 17 showed a significant superiority for the students who worked with the program, and only one showed a final superiority for the classroom students.[48]

Conflicting results concerning the relationship of intelligence to success in programmed instruction have been found. Feldhusen found that I.Q. was not a fundamental learning variable and that the variance in learning attributed to I.Q. seemed less important than that attributed to general achievement level.[49] However, using a program on modern mathematics for ninth graders, Lambert found intelligence to be significantly associated with the amount of information acquired from the program.[50]

In another kind of experiment Krumboltz compared required key-word response in programmed instruction with reading of the same material in paragraph form with no required response. He found no significant difference on post-tests. However, the subjects used in this study were college undergraduates and graduate students.[51] In a similar experiment Evans compared a programmed instruction text with a conventional text. Three topics were used - number bases, statistics, and music. There were higher post test scores for the students who used the programmed text, but these

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[48] Schramm, Op. cit., pp. 3-4.

[49] John F. Feldhusen and Andrew Birt, "A Study of Nine Methods of Programmed Learning Material," Journal of Educational Research, 55 (1962), 461-66.

[50] Phillip Lambert, Donald M. Miller, and David E. Wiley, "Experimental Folklore and Experimentation: The Study of Programmed Learning in the Wauwatosa Public Schools." Journal of Educational Research, 515, (1962), 485-94.

[51] John D. Krumboltz, The Nature and Importance of the Required Response in Programmed Instruction. Paper presented to American Educational Research Association, Chicago, February 1963 (Mimeographed.)

were significant only for music. The subjects in this experiment were also college undergraduates and graduate students.[52]

In an experiment of yet another type Garrison studied the effectivenss of programmed material as a supplement for elementary algebra over several ability levels. One group studied programmed materials in a teaching machine during class time for 15 minutes a day at their own rate. A second group, carried out the same study procedures, but did so during after school time. The third group, the control, continued in their usual fashion in the regular class. Each group contained all three ability levels, high, average, and low intelligence. Garrison found that the high ability groups used the machine programs more than the other two levels of ability did. He also found that the group which used regular class time for the program achieved better in the algebra, but that the difference was not significant at the five percent level.[53]

Summary

Research shows that students do learn from programmed instruction, but how it compares to other forms of instruction is still open to question. Experiments on the relationship of intelligence to success in learning from programmed instruction, and experimental comparisons between learning from programmed instruction and from conventional texts have been few and the results inconclusive.

IV. CONCLUSIONS

The difficulties of selecting talented students for special education and providing the right kind of program for them, taking into account the limitations imposed by the size of the school, have been made quite apparent by this review of the literature. Opinion seems to favor

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[52]James L. Evans, Robert Glaser, and Lloyd E. Homme, "An Investigation of Variation in the Properties of Verbal Learning Sequences of the 'Teaching Machine' Type." Teaching Machines and Programmed Learning: A Source Book, Lumsdaine and Glaser, Editors, (Washington, D.C., Department of Audio-Visual Instruction, National Education Association, 1960), 446-51.

[53]Nelson Garrison, Effectiveness of Programmed Material as a Supplement for Elementary Algebra Over Several Ability Levels.

enrichment within the regular class even though this method is as yet unproven. The same can be said of programmed instruction - although it is popular, its superiority is unproved. A basic program of enrichment, with materials or activites varied to suit the different types of potentially talented mathematics students is needed. An experimental study of various materials or activities such as programmed text versus conventional text may indicate the kind of enrichment differentiation needed. Although the enrichment program proposed in this study would be somewhat restrictive, it is intended merely as a basic program for the small junior high school and other enrichment activities recommended in the literature could still be adopted if they were considered successful and feasible for the particular school or class.

CHAPTER III

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

The purpose of this study was to design a practical mathematics enrichment program for the small junior high school. The program consisted of individual study of enrichment materials during regular mathematics class periods. Since the underlying philospphy of the program was that all potentially talented students should have some kind of enrichment experiences, two different types of potentially talented students were studied. The two types of students were compared on two different kinds of enrichment materials, programmed and conventional. In addition to this, the effects of this program on the regular class mathematics achievement of the students was determined.

This chapter contains a detailed description of the sample and materials used in the experiment, the design of the mathematics achievement tests, the design of the experiment, and the statistical procedures followed in processing the data.

I THE NATURE OF THE SAMPLE

One aspect of the experiment was to utilize a number of categories of potentially talented mathematics students. For this purpose, Hlavaty's categories were examined and modified.

Category D, consisting of those students whose socio-economic background hinders their school performance, was eliminated for two reasons. First, it was anticipated that the numbers of students in this category would be small compared to the numbers in the other categories. Because of the small size of the population in the first place, it was felt enough students of this type would not be available. Secondly, the description of the students in this category did not suggest to the investigator an enrichment treatment that was likely to be fruitful in a period as short as eight weeks.

Categories A and C consist of students of high general ability and mathematics achievement. However, they differ in that category A students exhibit all around high academic achievement, whereas category C do not.

Since the experiment was limited to a determination of outcomes in mathematics learning, the differences in achievement in other subjects between these two categories would have no significance in this experiment. Because of their similarities, these two categories were combined into one classification, Level I.

Category B, consisting of students of high general ability but mathematics achievement not commensurate with this ability, was adopted as Level II. It is possible that some category D students would be included in Level II. However, the numbers of such students would probably be negligible for the following reasons: no schools in the areas where these students were most likely to live were among those used in the experiment; about only one tenth of the population was slected for the Level II sample.

Permission was granted by the administration of the Edmonton Separate Schools to use pupils of their system in the experiment. All the grade seven mathematics teachers of the system were asked to cooperate and submit a list of all their students with an I.Q. of 115 or more. A few teachers declined to take part because their classes were considerably behind in coverage of regular curriculum content. Two more classes could not join in the experiment because they were special classes containing no students in the categories required and two were used to write the pilot study versions of the tests. Several more classes were eliminated because the teachers did not return the pre-tests before the deadline imposed by the school board. Hence, of the thiry seven grade seven classrooms of the system, nineteen finally participated as the experimental population from which the sample was drawn. This amounted to 355 students about two-thirds of the total population of seventh graders with I.Q.'s of 115 or more in the system.

The size of the sample was determined by a procedure outlined by Winer[1],

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[1] B.J. Winer, <u>Statistical Principals in Experimental Design</u>, (New York: McGraw-Hill Book Company, 1962), p. 104.

booklet provided satisfactory horizontal enrichment that could be made to mesh smoothly with the regular grade seven mathematics curriculum. In addition, each series as a whole provided adaquate vertical enrichment. Some indication of these facts can be seen by an examination of table I, page 32. This table shows the chapter headings and sub-headings of the regular grade seven mathematics curriculum text along with the titles of each booklet in each enrichment series arranged in order to suggest their possible time of use so that they would mesh smoothly with the regular curriculum. The conventional material was the more extensive of the two series. However, the publisher plans many more texts in the programmed series which would be suitable for junior high school mathematics enrichment. Since the students in the experiment would be studying numeration and number systems in the regular curriculum during the experimental period, Modular Systems[3] and Finite Mathematical Systems[4] were chosen as the programmed and conventional texts respectively.

For the purpose of this study the only desired difference between the texts was that one should exhibit common characteristics of programmed material while the other should represent conventional mathematic text format. It was important that the texts be similar in readability, content coverage, notation, amount of practice provided, difficulty level of exercises, and basic approach to the development of concepts. There may be other minor differences between the texts which could influence the results such as general appearance, size of type,and so on. It was felt that these differences, if they existed, would not constitute a major limitation of the experiment. It was assumed that any other important differences between the texts could be attributed to the differences in format, programmed versus conventional, which would show up in the experiment.

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[3]Edward J. Zoll, Modular Systems - An Introduction to Structure in Mathematics, (London: Collier-MacMillan Limited, 1965).

[4]Norton, Op. cit.

the calcuation of which is shown in Appendix C. This amounted to approximately 20 students per cell. Since there were three treatment groups and two categories, the total number required for the experiment was 120--sixty for each category. On the basis of the pre-tests covering the mathematics taken up so far during that year, the top 60 and the bottom 60 were selected and categorized as Level I and Level II respectively.

II. THE NATURE OF THE MATERIALS

The review of the literature suggested that one aspect of the experiment consist of comparing the suitability of programmed and conventional mathematics enrichment materials for each category. This meant that a pair of such enrichment texts covering the same content and meshing smoothly with the regular mathematics curriculum had to be found. Interest in establishing a complete grade seven enrichment program led to the desire to find not a single pair of texts, but two series of texts. Since it was discovered that there were fewer programmed texts available, it was decided to examine these first.

The *Understanding Modern Mathematics* series of programmed texts was the only one the investigator could find that met the conditions described above. In the conventional form, the most comparable series was found to be *Exploring Mathematics on Your Own*. The particular suitability of this series of texts for the type of enrichment program used in this experiment is expressed by the editors in the following way:

> Exploring Mathematics on Your Own is a fascinating series of enrichment booklets tailored for students who want to go beyond the textbook. Each booklet in the series presents a stimulating topic in an informal, easy-to-read style. Each booklet is complete in itself and may be used by students for individual work or by the class as a whole. Ample practice is provided.[2]

Examination of both series led the investigator to believe that each

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[2] M. Scott Norton, *Finite Mathematical Systems*, (St. Louis: Western Division, McGraw-Hill Book Company, 1963).

TABLE I

CHAPTER HEADINGS AND SUB-HEADINGS FOR THE SEEING THROUGH MATHEMATICS, UNDERSTANDING MODERN MATHEMATICS, AND EXPLORING MATHEMATICS ON YOUR OWN TEXT SERIES MATERIALS FOR GRADE SEVEN

Seeing Through Mathematics, Book I	Understanding Modern Mathematics	Exploring Mathematics on Your Own
1. Sets, conditions, and variables -- finite and infinite sets, conditions, problem solving, sets of points, such as lines, planes, and circles	1. Points, Lines and Planes 2. Number Sentences	1. Geometric Constructions * 2. Invitation to Mathematics 3. Basic Concepts of Vectors
2. Intersection and Union of Sets -- intersection and union of sets, compound conditions, problem solving, intersections of lines and planes, rays, angles, triangles	3. Points, Lines and Planes 4. Numbers Sentences	4. Geometric Constructions 5. Basic Concepts of Vectors 6. Sets, Sentences and Operations
3. Condition in two variables--sets of ordered pairs, graphs, problem solving	5. Number Sentences	7. Sets, Sentences and Operations 8. Adventures in Graphing
4. Conditions involving rate pairs--rate pairs, percent, business arithmetic, problem solving	6. Number Sentences 7. Bases and Numerals	9. Short Cuts in Computing 10. Understanding Numeration Systems
5. Numeration Systems - numeration systems in bases other than ten	8. Bases and Numerals 9. Modular Systems	11. Computing Devices 12. Understanding Numeration Systems 13. Finite Mathematical Systems

6. The natural number system --operations and properties of the operations of the natural numbers, composite numbers, prime numbers

7. The rational numbers of arithmetic -- rational numbers, operations and properties of the rational numbers, conditions, decimal fraction numerals, problem solving.

10. Modular Systems

11. Factors and Primes

12. Number Sentences

13. Modular Systems

14. Factors and Primes

*15. What Are the Chances (probability)

14. Number Patterns

15. Finite Mathematical Systems

* 16. Logic & Reasoning in Mathematics

* 17. Fun with Mathematics

* 18. Curves in Space

* 19. World of Statistics

* 20. Probability and Chance

* 21. The World of Measurement

* 22. The Pythagorean Theorem

* 23. Topology - the rubber sheet geometry.

*
Can be used anytime or in grade 8

The readabilities of the programmed and the conventional texts were found to be grade levels 8.0 and 8.5 respectively. The investigator felt that these readability levels were comparable and suitable for the students in the experiment because they were above average I.Q. students nearing the completion of their grade seven year. The computations of the readabilities are shown in Appendix C.

A comparison of the content of the two texts is shown in Table II, page 35. Examination of this table reveals to some extent that content is comparable up to a point. The main difference in the content coverage of these two texts was that the conventional text covered more. Since this extra content occurred toward the end of the text, it was decided to eliminate that section of the conventional text from the experiment and use only the portion up to Non-Prime Modular Systems. A further indication of the comparability of content coverage of the two texts is indicated by the introductory remarks in each:

> Modular Systems
>
> In your studies in mathematics you have probably learned that at various times in history people have written numbers in different ways. certainly you are familiar with roman numerals, and it is not at all surprising that the Egyptians, Hindus, Greeks, Mayans, and many other peoples had their own particular systems of numberation. It is important to recognize that all these various systems were simply different ways of writing or recording the same, fundamental ideas about numbers. That is, when the various peoples added, say, 14 and 5, they always found the sum to be 19, although they may have written this fact with very different-looking symbols.
>
> Now, in this book you are going to learn about some mathematical systems that are quite different from the one you have been studying in arithmetic. For example, in one of these systems you will find that 2 + 4 = 1 and 3 - 4 = 4. In another of these systems it turns out that 4 divided by 2 is equal to either 2 or 5! Fortunately, it will not be necessary for you to memorize new sets of number facts, because most of the time that you are working with these new systems you will be using present knowledge of arithmetic.
>
> In the first chapter you will start with an idea that you have been using for some time--remainders in division problems. You will keep using this idea throughout most of this program, although you will learn some new ways of writing remainders. Then you will discover some unusual properties of "remainder numbers".

TABLE II

PROGRAMMED AND CONVENTIONAL TEXT CHAPTER HEADINGS

<u>Programmed</u>

<u>Modular Systems</u> by R.J. Zoll

1. REMAINDER NUMBERS

    Number Discoveries and Remainders

    Using Remainder Sets

    Adding Remainder Sets

    Subtracting Remainder Sets

    Subtraction Revisited

2. IDENTITY AND INVERSE ELEMENTS

    Modular Elements

    Additive Identity Elements

    Additive Inverse Elements

    Addition Revisited

    Additions such as $1^- = 2^-$

    Subtraction Revisited

    Subtracting Additive Inverse Elements

    Subtractions Such as $1^- - 2$

    Subtractions Such as $2^- - 4^-$

3. MULTIPLICATION AND DIVISION

    Modular Multiplication

    Modular Division

    Division by Zero

    Multiplication Using Additive Inverse Elements

    Division Using Additive Inverse Elements

<u>Conventional</u>

<u>Finite Mathematical Systems</u> by M. Scott Morton (Annotated)*

1. EXPLORING FINITE SYSTEMS IN MATHEMATICS

    Mathematics is the Exact Science -- or Is It?

    Seven Days Make A Week -- and also an Interesting Finite Arithmetic System
    * (residue classes using clock arithmetic approach to addition and subtraction)

2. FINITE SYSTEMS AND PROPERITES OF MATHEMATICS

    Certainly 7 + 6 = 6 + 7 -- or Does It?
    * (commutative property)

    The Association Property -- an Important Idea

    Other Important Properties -- Identities, Inverses, and Distributivity

3. NEW DISCOVERIES IN MOD 7 ARITHMETIC

    Multiplication in Finite Arithmetic

    Applications of Modular Multiplication

4. EXPLORING OTHER OPERATIONS IN FINITE SYSTEMS

    Eliminating the Negative

    Are Fractions Necessary

    Proceed with Caution Especially When Working With Zero *(division by zero)

    A Strange New Clock Arithmetic

5. NON-PRIME MODULAR SYSTEMS
   -- NEW VIEWPOINTS

   Finite Mathematical Systems
   Simplify Matters -- or Do They?

   Keep Working and You will Find
   the Answer -- Not Always!

6. New Symbols and New Structures
   -- (abstract finite systems)

7. Finite Geometry - a new
   Consideration (axiomatic
   approach)

*Annotated notes on Finite Mathematical Systems appear in parentheses. These have been added because the sub-headings are often not descriptive of the content covered.

Sometimes you will find these new systems to be easy, other times, perplexing. You will start with a very easy idea about reaminders in division problems and then find what it means to add, subtract, multiply, and divide with these new "remainder numbers". In this way you will be taking a few more steps into the "world of mathematics".[5]

Finite Mathematical Systems

So you think you know arithmetic? How much is 4+5? if you answer 9, you could be wrong! There is an arithmetic system in which 4 + 5 = 2. Other curious arithmetic facts in this system are 3 + 4 = 0, 5 ÷ 3 = 4, and 4 X 5 = 6. This is only one of the systems of arithmetic that you will study in this booklet. If such mathematics systems seem strange to you, you may be surprised to learn that you use systems similar to the one mentioned above several times a day. It can be called clock arithmetic, arithmetic of residue classes, or the study of finite mathematical systems.

Learning about finite mathematical systems is one of the most interesting studies in mathematics. In general, a finite system is one which has a specific number of elements. The number of elements must be zero or a counting number. This type of system is in contrast to the system of natural numbers which has an unlimited (infinite) number of elements.

Examples of sets with a finite number of elements are:

A dozen apples
The numerals on a clock
Your weight in pounds
The odd numbers between zero and twenty
Zero and one

Examples of sets with an infinite number of elements are:

The set of all fractions
The set of all squares
The number of lines in a plane

In a similar way, some mathematical systems are finite and others are infinite.

In this booklet, we will explore some ideas concerned with finite mathematical systems. Through exploring this topic, you will learn to know and to appreciate structure in mathematics. You will also develop a better understanding of the logic of mathematics and learn how to use this logic in your study of mathematics.[6]

These introductions also indicate one basic difference to the approach taken in developing the concepts of finite mathematical systems.

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[5] Zoll, Op. cit., p. 1.

[6] Norton Op. cit., pp. 2-3.

In the programmed text, Modular Systems, these concepts were based on the idea of remainder or residue classes, whereas in the conventional text the development was based on the idea of clock arithmetic. Hence, a possible limitation of the experiment is that, if one of these approaches was superior to the other, any differences in achievement between the experimental groups may have been attributable to this difference in approach. However, both texts did eventually include each others approaches as applications of finite systems.

Another difference in the texts which could confound the results was in notation. The most notable example of notational difference was in the manner of writing down exercises concerning modular operations. Extracts from problem sections on modular addition illustrate this point.

Modular Systems

1. In each problem below, the divisor is 6. Find the sums.

a. $[0]_6 + [3]_6 = [\ ]_6$ 3

b. $[5]_6 + [4]_6 = [\ ]_6$ 3

. .

. .

. .

j $[5]_6 + [2]_6 = [\ ]_6$ 1

2. Try filling in the missing part in each of these addition problems:

a. $[1]_6 + [\ ]_6 = [2]_6$ 1

b. $[2]_6 + [\ ]_6 = [5]_6$ 3

. .

. .

. .

j $[\ ]_6 + [4]_6 = [3]_6$ 5 [7]

Finite Mathematical Systems

1. On a regular 12-hour clock, what time is it 4 hours after 9:00 A.M.? Six hours after 7:00 P.M.?

On the regular clock, 9 + 4 = 1 and 7 + 6 = 1. By defining the operation of addition on the clock as rotating clockwise the number of spaces

[7]Zoll, Op. cit., pp. 18-19.

indicated by each addend, complete the following table in figure 5 for addition for the finite clock arithmetic system mod 12.

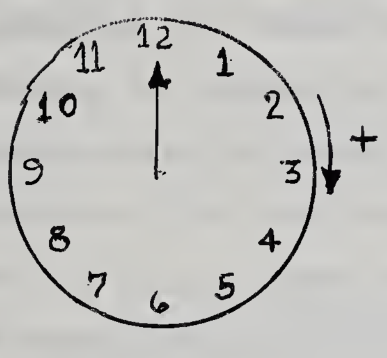

Figure 4

Addend

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	1
2	3	4										
3	4		6									
4				8								
5					10							
6						12						
7							2					
8								4				
9									6			
10										8		
11											10	
12												12

Addend

Figure 5

The difference in notation was that a subscript was used to indicate the modular number in Modular Systems. However, in both texts the following notation was also used: 7 + 6 = 1 mod 12. Since there was this common notation, and because the notational differences were pointed out by all the teachers to the experimental groups and mentioned on the final enrichment mathematics achievement test, it was felt that the notational difference would not bias the results in favor of one group or the other.

The examples cited above also indicate the comparability of the practice exercises provided in both texts. Both examples represent the entire sets of practice exercises provided following the topic of modular additions. Figure 5 in the <u>Finite Mathematical Systems</u> exercise required the student to do 110 modular additions. However, the topic of the commutative

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[8]Norton, <u>Op. cit.</u>, pp. 6-7.

property of addition was also taken up prior to these exercises, so if the student had learned this concept well, he would have had to do only 55 additions. The most superior students who were accustomed to looking for number patterns may have been able to fill in the table without doing any actual additions. The exercises in Modular Systems required the students to do only 20 additions. However, there were about 56 additions required in the actual text development of the concept of modular addition which preceded the exercises, compared to only a few in the other text development. Hence, the amount of practice provided in both texts, as shown by this representative example, was about the same, except that in this particular example, the most superior students using the conventional text may have had to do far fewer exercises. This difference was thought to be an inherent characteristic in the difference between programmed and conventional format rather than a difference in the amount of practice provided. Because the development of the concept of modular addition was broken down into small sequential steps with immediate reinforcement, it was necessary to require many modular additions during the development of the topic. However, in the conventional text the concept was merely explained in paragraph form with a few examples. The practice was left mainly to the exercises with answers provided at the back of the book.

In assessing the difficulty level of the exercises it was found that, where arithmetic operations were involved, the conventional text exercises appeared to be easier. The investigator felt that performing operations using the clock and table as an aid as was done in the conventional text was easier than performing ordinary arithmetic operations to find remainders as was required in the programmed text exercises. However, since the operations generally involved relatively simple computations, and since all the students in the experiment were at least average in previous mathematics achievement, it was felt that the advantage of the conventional text over the programmed text on this point was very slight. A subjective evaluation of other kinds of exercises such as problem solving led the investigator to believe that the difficulty levels of the exercises of the texts were comparable.

The examples of exercises shown on pages 38 and 39 also provide a further indication of the differences in basic approach as mentioned before -- remainder numbers and clock arithmetic. However, as indicated before, each text includes the approach of the other in later sections as applications.

In the programmed text, Modular Systems, most sections begin with a brief introduction to the topic, and then continue the development with typical programmed frames, whereas in the convnetional text, Finite Mathematical Systems, the development is in the form of explanatory paragraphs with examples and occasional unanswered questions. Examination of the following excerpts on the development of the idea of the additive identity element in each text serves as a good illustration of these points:

Modular Systems

Note: In a modular system we can call the remainder numbers elements of a set. For example, in mod 5 the modular elements are 0,1,2,3, and 4. We can call these the elements for mod 5.

11. Let's look at a new idea that Ellen, Ed, and Tom explored. The idea involved the element 0 when it is used in modular addition. The three students found the answer to each of these addition problems in mod 5.

$0 + 0 =$ ______ (mod 5) — 0

$0 + 1 =$ ____ (mod 5)	$1 + 0 =$ ____ (mod 5)	1,1
$0 + 2 =$ ____ (mod 5)	$2 + 0 =$ ____ (mod 5)	2,2
$0 + 3 =$ ____ (mod 5)	$3 + 0 =$ ____ (mod 5)	3,3
$0 + 4 =$ ____ (mod 5)	$4 + 0 =$ ____ (mod 5)	4,4

12. If $\triangle$ represents any mod 5 element and

$$\triangle + 0 = \triangle \pmod 5$$
$$\text{and} \quad 0 + \triangle = \triangle \pmod 5$$

we say that 0 is the identity element for mod 5 addition. (the sum of zero and any element is identical with the element.)

Let's see if 0 is the identity element for mod 4 addition.

$0 + 0 =$ ______ (mod 4) — 0

$0 + 1 =$ ____ (mod 4)	$1 + 0 =$ ____ (mod 4)	1,1
$0 + 2 =$ ____ (mod 4)	$2 + 0 =$ ____ (mod 4)	2,2
$0 + 3 =$ ____ (mod 4)	$3 + 0 =$ ____ (mod 4)	3,3

13. In mod 4 addition, the sum of zero and any element is <u>identical</u>. with_____?_____.

a. 0 b. the element c. 4 b. the element

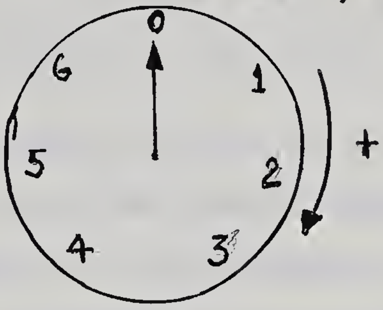
14. Then we can say that 0 is the additive________ identity [9]

<u>Finite Mathematical Systems</u>

We have discussed earlier the fact that 7 added to the number which represents any day of the week, such as Sunday, returns you to that same day.

In our system of clock arithmetic mod 7, seven has been replaced by 0.

Figure 7



In all cases as 6 + 0, 5 + 0, 4 + 0, 3 + 0, 2 + 0, 1 + 0, and 0 + 0, the sums are identical to the element which was added to 0. We say that 0 is the <u>identity element</u> for the addition operation, since for any element n, n + 0 = n. This is an interesting and important property to consider in the structure of a mathematical system. We will need to consider this property in more detail as we explore other finite systems.

Can you name the identity element for addition in the system of ordinary arithmetic? 13+ <u>?</u> = 13; <u>?</u> + 23 = 23?[10]

The main difference between these two developments of the concept, apart from that of notation and that of basic approach (remainder numbers versus clock arithmetic) was that in the programmed text the students were forced to respond to a development in small sequential steps with immediate reinforcement, whereas in the conventional text the ideas were explained to the student and followed by questions to which the answer often was not provided. The programmed text also seemed to provide the opportunity for a type of discovery. In this case, as illustrated in frame 11, the students were required to respond to a number of questions based on previous learned knowledge. These questions were arranged so as to provide the opportunity for the student to discover the concept by recognizing a pattern. Only after this was the concept explained. A similar kind of opportunity for

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[9] Zoll, <u>Op</u>. <u>cit</u>., pp. 35-36.

[10] Norton, <u>Op</u>. <u>cit</u>., p. 12.

discovery, although less obvious, was sometimes apparent in the conventional text. An example of this was the pattern in the table used for the exercise on modular addition which was cited previously. However, since this type of discovery opportunity was used continuusly throughout the programmed text, it was considered an inherent characteristic of that type of format, and a basic point of difference with the conventional text.

III. DESIGN OF THE TESTS

This section contains a description of the design of the three tests prepared by the investigator, the Pre-Test of Regular Mathematics Achievement, the Post Test of Regular Mathematics Achievement, and the Enrichment Mathematics Achievement Test, referred to hereinafter as the PRMA, PMA, and EMA tests respectively.

The PRMA and PMA Tests

The PRMA and PMA tests were intended to reflect the objectives of the regular classroom text and of the schools. It was learned that, since this was the first year of the new course, most teachers were following the text quite closely and using mainly the exercises and tests from the text. The teacher's guide contained a number of supplementary tests. Some of the items in these tests, particularly on geometry, had been used in teacher-made tests. Therefore, in order to feflect the objectives of both the authors and the teachers, most of the PRMA and all of the PMA items were chosen from among those of the supplementary tests which had not been used by any of the teachers. The items in the PRMA which were not chosen from among the supplementary tests were included because some of the teachers had indicated that the supplementary test items did not discriminate well between the average and the above average students. These extra items represented number concepts which should be understood by average and above average students in varying degrees, and were chosen in consultation withsome of the teachers.

Pilot study versions of both tests were administed to 36 students with I.Q.'s of 115 or more who were not included in any other part of the experiment. The tests were discussed in a seminar at the university

by the investigator's advisor, three graduate students in mathematics education, and the investigator, and also, on another occasion by several grade seven mathematics teachers and the investigator. As a result of these discussions and an analysis of the pilot study version test data, several items of the original 50 in each test were eliminated or improved. After the 40-item final versions of the tests had been administered, all the teachers involved in the experiment were asked to give their opinions of the tests. The teachers indicated that the tests did reflect the objectives of the course as taught by them. The reliabilities of the final versions of the PRMA and PMA tests as calculated by the Kudner-Richardson 20 formula were found to be .77 and .90 respecitvely. The calculation of these reliabilities along with the final versions of each test, and difficulty and discrimination indices for each item of the pilot versions of the tests are included in Appendix B.

The EMA Test

This test was designed to cover all areas of common content in both the enrichment texts. Care was taken not to introduce bias in favor of one or other of the experimental groups in the selection or writing of the test items. Although the students were aware of notational differences, wherever this difficulty arose in the rest the differences were carefully explained. It was not possible to administer a pilot study version of this test as there were no students with knowledge of the content available. However, the same expert advise as in the cases of the PRMA and PMA tests was sought and used. In order to reflect the objectives of the authors of the texts, the items used were modifications of those in the enrichment texts. The reliability of the test as calculated by the Kuder-Richardson formula 20 was found to be .72. The calculation of this reliability index, difficulty and discrimination indices, and a copy of the EMA test are included in Appendix B.

IV. EXPERIMENTAL DESIGN AND DATA COLLECTION

Control of Variables

The following variables, which includes those mentioned by Hlavaty,[11] were considered factors which might influence the students' PMA and EMA scores: sex, teacher influence, chronological age, reading ability, achievement in academic subjects other than mathematics, I.Q., previous achievement in mathematics, and attitude toward mathematics. All of these factors were experimentally controlled by means of random assignment to the control and treatment groups.

It was considered unnecessary to exercise further control over the first five variables. Examination of Tables III, IV, and V on pages 46, 47 and 48, reveals the reasonable comparability of the experimental groups on sex, teacher influence and chronological age factors. There were two reasons why reading ability was not controlled further. First, the readabilities of the programmed and conventional texts were comparable and close to the grade level of the students in the experiment. Second, there is evidence that reading ability is highly correlated with I.Q., a variable over which further control was to be exerted.[12] Because of the delimitation of the experiment to measurement in terms of mathematical outcomes, and in view of what had been said in the literature on the unreliability of school marks as a basis of predicting success in enrichment, it was considered beyond the scope of this experiment to consider academic achievement in subjects other than mathematics.

The literature indicated that I.Q., previous mathematics achievement, and attitude toward mathematics were important factors to consider. Therefore, these three variables were further controlled by using the students' scores on these as covariates in analyses of covariance with PMA and EMA acores as criteria.

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[11] See Chapter II, p. 14.

[12] Monroe, Op. cit., pp. 1103-1104.

TABLE III

DISTRIBUTION OF BOYS AND GIRLS FOR EACH LEVEL OVER TREATMENT GROUPS

TREATMENT	CONVENTIONAL		PROGRAMMED		CONTROL	
LEVEL	BOYS	GIRLS	BOYS	GIRLS	BOYS	GIRLS
I	9	11	11	9	9	11
II	11	9	10	10	10	10
TOTAL	20	20	21	19	19	21

TABLE IV

NUMBER OF STUDENTS ASSIGNED TO EACH TEACHER OVER TREATMENT GROUPS

TREATMENT	TEACHER																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
CONVENT-IONAL	5	6	2	1	3	5	1	5	2	1	3	2	1	1	1	1	0	0	0
PROGRAM-MED	4	4	1	1	5	1	1	3	2	2	3	2	1	3	2	1	1	2	1
CONTROL	4	4	0	2	4	3	2	3	0	2	1	1	2	1	4	3	1	3	0

TABLE V

MEAN CHRONOLOGICAL AGE IN YEARS AND MONTHS FOR EACH LEVEL OVER THE TREATMENT GROUPS

LEVEL	TREATMENT		
	CONVENTIONAL	PROGRAMMED	CONTROL
I	12 - 8	12 - 9	12 - 10
II	12 - 10	12 - 8	12 - 10

[illegible]

[illegible]

[illegible]			[illegible]
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Testing Program and Experimental Procedure

The PRMA test was administered to 355 grade seven students during February, 1966. These students were considered representative of the total population of seventh grade mathematics students with I.Q.'s. of 115 or more of the Edmonton Separate School System. Examination of Table VI confirms this fact. This table is a description of the I.Q. and mathematics achievement of the total population, the population from which the sample was drawn (the 355 students), and the sample (the 120 students). The mathematics achievement data were taken from a School Board mathematics Easter examination which was administered to all the grade seven mathematics students in the School System in April 1966. This was a 3-hour test which covered all the grade seven regular curriculum mathematics taken by the students up until April.

The top and bottom 60 students on the PRMA were classified as Level I and II respectively. For each level separately, 20 of the 60 students were assigned randomly to each of the two treatments and the control. One treatment group was assigned to study Finite Mathematical Systems and the other Modular Systems. The studying of these enrichment materials was done during one thirty-minute regular mathematics period each week for eight weeks. The students were required to study the enrichment materials entirely on their own and only during these periods. During these eight periods the control group continued with regular mathematics classwork. At the end of the experimental period, in April, the PMA and EMA tests were administered.

The I.Q. scores of all students were taken from the Otis Alpha Quick Scoring Test which had been administered to them in grade five. In the Fifth Mental Measurements Yearbook,[13] this test is reported as having a reliability of .97. Evidence of the validity of the test is

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[13]Curt Buros, (editor) Fifth Mental Measurements Yearbook, (the Gryphon Press, Highland Park, New Jersey, 1965,) p. 481.

TABLE VI

COMPARISON OF I.Q. AND MATHEMATICS ACHIEVEMENT FOR THE TOTAL POPULATION POPULATION FROM WHICH SAMPLE WAS DRAWN, AND SAMPLE

STATISTIC	TOTAL POPULATION (560 students)	POPULATION FROM WHICH SAMPLE DRAWN (355 students)	SAMPLE (120 Students)	
			Level I	Level II
I.Q. Mean	123.11	122.12	125.69	118.62
Standard Deviation	6.33	5.58	5.12	6.14
Mathematics Mean	112.93	114.70	120.08	103.21
Standard Deviation	17.26	18.74	16.42	14.61

also presented. The test is described as essentially a scholastic ability test. Although this test had been administered two years prior to the experiment, it was considered practical to use these scores. No change in the System's policy of not administering I.Q. tests in grades six and seven was anticipated. Therefore, it was felt that, since any enrichment practice adopted by the System would have to be reliant on grade five I.Q. scores, these should also be used in the experiment.

Remai's attitude scale,[14] referred to hereinafter as the MAS, was used as the measure of the students' attitude toward mathematics. This scale was considered particularly appropriate because some Alberta junior high school mathematics students using the same new mathematics curriculum as the students of the Edmonton Separate School System were used to test the validity and reliability of this instrument. The scale is a multiple choice type. There are five possible completion responses to each item. The response indicating the most favorable attitude is assigned a weight of 5, and the least favorable response is given a weight of 1. A student's total score on the scale represents the sum of the weights assigned to his responses to each item. The highest possible score is 115 and the least is 23. This scale is reported to have a concurrent validity of .43, acceptable content validity based on expert opinion, test-retest reliability of .77, and internal consistency of .86.

At the conclusion of the experimental period, the teachers who were involved in the experiment were asked to complete a questionnaire about their opinions on the enrichment procedures used. In chapter IV, a summary of the teachers' opinions is presented.

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[14]Henry Albert Remai, " An Experimental Investigation Comparing Attitudes Toward Mathematics of Modern and Traditional Mathematics Students at the Junior High School Level," (Unpublished Master of Education Thesis, University of Alberta, 1965).

V. NULL HYPOTHESES

The data collected and described in the preceding pages were used to test the following null hypotheses statistically:

1. With seventh grade students, there is no significant difference at the five percent level in regular mathematics classwork achievement among the control and experimental groups within
   (a) Level I,
   (b) Level II,
   (c) Levels I and II combined.
2. With seventh grade students, there is no significant difference at the five percent level in enrichment mathematics achievement among control and experimental groups within
   (a) Level I,
   (b) Level II,
   (c) Levels I and II combined.

VI. STATISTICAL PROCEDURES

Each of the 120 students in the sample was assigned an identification number, and his I.Q., MAS, PRMA, PMA, and EMA, and his Identification Number were punched on an IBM card. A complete listing of this information for each student is included in Appendix A. These cards were then processed by the IBM 604 computer as described in the succeeding paragraphs.

Since it was decided to control the effects of previous mathematics achievement, attitude toward mathematics, and I.Q., one-way analyses of covariance with PRMA, I.Q., and MAS scores as covariates and PMA and EMA scores as criteria wereconsidered appropriate for testing the two hypotheses. The procedure used was an adaption of that recommended by Winer.[15] The computation formulas used for this analysis are presented in Appendix C. The Newman-Kuels procedure[16] was used to make comparisons between ordered means.

For both null hpotheses, tests among the three groups, the two treatments and the control, were performed within Levels I and II combined.

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[15]B.J. Winer, Statistical Principles in Experimental Design, (New York: McGraw-Hill Book Company, 1962), pp. 618-621.

[16]Ibid., pp. 80-84.

The purpose of combining these two levels for the tests was to determine the usefulness of the Level I and Level II categorizations. If the results for the two levels separately differed from the results of the two levels combined, it would be considered that the categorization of potentially talented mathematics students into two levels had been a useful procedure.

Homogeneity of variance was tested by means of the Hartley tests and found to be satisfactory. Another assumption is that the distributions of the variables in the populations from which the samples are drawn are normal. According to Johnson and Jackson, this is assumed to be true if the sample used is large, since "the distribution is amazingly unaffected by lack of symmetry."[17] In this experiment the sample was large compared to the size of the population from which it was drawn.

Certain additional assumptions underlie the analysis of covariance. In describing these Winer says,

> First, it is assumed that treatment effects and regression effects are additive. Implicit in this is the assumption that regressions are homogeneous. Second, it is assumed that the residuals are normally and independently distributed with zero means and the same variance. Implicit in this is the assumption that the proper form of regression equation has been fitted.[18]

Inspection of the data led the investigator to believe that these additional assumptions were satisfied.

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[17]Palmer O. Johnson, and Robert W.B. Jackson, _Modern Statistical Methods: Descriptive and Inductive_, (Chicago: Rand McNally & Company, 1959), pp. 193-194.

[18]Winer, _Op. cit._, p. 586.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

I. FINDINGS FROM THE STATISTICAL ANALYSIS

In this section the results of the analysis of the data are presented. Each null hypothesis to be tested is stated and followed by the appropriate statistical test and a brief interpretation. The section is concluded with a summary of the results.

<u>Analysis of Covariance, PMA Test Scores</u>

The procedures followed in carrying out the one-way analysis of covariances are presented in Appendix C. The results of the analysis are presented following the null hypothesis.

<u>Null Hypothesis I</u>

> With seventh grade students, there is no significant difference at the five percent level in regular mathematics classwork achievement among the control and experimental groups within
> a) Level I,
> b) Level II,
> c) Levels I and II combined.

The PMA test cell mean scores adjusted and unadjusted for the effects of I.Q., PRMA, and MAS scores are listed in Table VII, page 55. A summary of the analysis of covariance on the PMA scores with I.Q., PRMA, and MAS scores as covariates is presented in Table VIII, page 56. The abbreviation SS stands for "Sums of Squares" and means "the sum of squares of the difference between each score and its particular group mean." The symbol MS stands for the "mean square" which is the average variation per degree of freedom." The abbreviation df stands for "degrees of freedom". F represents the F-ratio which is obtained by dividing the group MS by the error MS. The level of the probability at which the F-ratio is significant is represented by the letter p.

TABLE VII

PMA CELL MEANS ADJUSTED AND (UNADJUSTED) FOR THE EFFECTS OF I.Q., PRMA, AND MAS SCORES

	CONVENTIONAL	PROGRAMMED	CONTROL
LEVEL I	29.49 (29.55)	28.29 (28.65)	30.02 (29.60)
LEVEL II	16.82 (17.15)	17.37 (17.13)	20.33 (20.25)
COMBINED	22.84 (23.35)	23.08 (22.89)	25.25 (24.93)

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

TABLE VIII

SUMMARY OF ANALYSES OF COVARIANCE: PMA SCORES ADJUSTED FOR THE EFFECTS OF I.Q., PRMA, AND MAS SCORES

a) LEVEL I

SOURCE	SS	df	MS	ADJ F	P
GROUP	29.75	2	14.88	.33	.715
ERROR	2383.10	54	44.13	$F_{.95}(2.54) = 3.14$	

b) LEVEL II

SOURCE	SS	df	MS	ADJ F	P
GROUP	130.78	2	65.39	2.71	.076
ERROR	1305.18	54	24.17	$F_{.95}\ (2,54) = 3.14$	

c) LEVELS I AND II COMBINED

SOURCE	SS	df	MS	ADJ F	P
GROUP	136.14	2	68.07	1.88	.158
ERROR	4137.06	114	36.29	$F_{.95}\ (2,114) = 3.08$	

In each case the observed F-ratio for the comparisons among group means on the PMA scores did not exceed the critical values. Hence, the results of these analyses led to the following decisions:

Null Hypothesis 1 a) was <u>accepted</u>,

Null Hypothesis 1 b) was <u>accepted</u>,

Null Hypothesis 1 c) was <u>accepted</u>.

These results indicate that students of both Level I and Level II were able to use regular class mathematics periods for enrichment study without loss in regular classroom achievement.

<u>Analysis of Covariance, EMA Test Scores</u>

<u>Null Hypothesis 2</u>

With seventh grade students, there is no significant difference at the five percent level in enrichment mathematics achievement among the control and experimental groups within
a) Level I,
b) Level II,
c) Levels I and II combined.

The EMA test cell mean scores adjusted and unadjusted for the effects of I.Q., PRMA, and MAS scores are listed in Tables IX, page 58. A summary of the analysis of covariance on the EMA scores with I.Q., PRMA, and MAS scores as covarieates is presented in Table X, page 59. The observed F-ratio for comparisons among group means on the EMA scores did not exceed the critical value for Level II. However, the observed values did exceed the critical values for Level I and for both levels combined. Hence, the results of these analyses led to the following decisions:

Null Hypothesis 2 a) was rejected,

Null Hypothesis 2 b) was accepted,

Null Hypothesis 2 c) was rejected.

Since differences among adjusted group mean EMA scores were statistically significant for Level I and for both levels combined, further comparisons among the group means were made by means of the Newman-Keuls procedures. In Table X, page 59, the three group EMA adjusted cell means for Level I and for the two levels combined

TABLE IX

EMA CELL MEANS ADJUSTED AND (UNADJUSTED) FOR THE EFFECTS OF I.Q., PRMA, AND MAS SCORES

	CONVENTIONAL	PROGRAMMED	CONTROL
LEVEL I	15.95 (16.03)	14.27 (14.33)	11.73 (11.60)
LEVEL II	10.57 (10.65)	10.33 (10.40)	10.40 (10.93)
COMBINED	13.14 (13.34)	12.47 (12.36)	11.02 (10.93)

[illegible]

[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	

TABLE X

SUMMARY OF ANALYSES OF COVARIANCE: EMA SCORES ADJUSTED FOR THE EFFECTS OF I.Q., PRMA, AND MAS SCORES

a) Level I

SOURCE	SS	df	MS	ADJ F	P
GROUP	173.94	2	86.97	16.06*	.000
ERROR	292.68	54	5.42	$F_{.95}(2,54) = 3.14$	

b) Level II

SOURCE	SS	df	MS	ADJ F	P
GROUP	.58	2	.29	.04	.956
ERROR	356.94	54	6.61	$F_{.95}(2,54) = 3.14$	

c) Levels I and II Combined

SOURCE	SS	df	MS	ADJ F	P
GROUP	91.72	2	45.86	6.65*	.002
ERROR	786.60	114	6.90	$F_{.95}(2,114) = 3.08$	

* Significant

TABLE XI

TESTS ON THE DIFFERENCES BETWEEN ORDERED GROUP CELL MEAN SCORES ON THE EMA TEST

a) Level I

GROUP		CONTROL	PROGRAMMED	CONVENTIONAL
	MEANS	11.73	14.27	15.95
CONTROL	11.73	--	2.54* (1.48)	4.22* (1.77)
PROGRAMMED	14.27		--	1.68* (1.48)
CONVENTIONAL	15.95			--

b) Levels I and II Combined

GROUP		CONTROL	PROGRAMMED	CONVENTIONAL
	MEANS	11.02	12.47	13.14
CONTROL	11.02	--	1.45* (1.18)	2.12* (1.41)
PROGRAMMED	12.47		--	.67 (1.18)
CONVENTIONAL	13.14			—

are ordered from least to greatest as the table is read from left to right or top to bottom. The two entries in each cell of the table represent the absolute difference between the groups being compared and, enclosed in parentheses, the critical five percent level difference. If the observed absolute difference exceeds the critical value, the difference in the means being compared is considered statistically significant. Procedures used in performing this analysis are presented in Appendix CV.

For Level I all three comparisons between pairs of means were found to be significant. In other words, the adjusted mean score on the EMA test for the conventional group was significantly higher than the mean scores for either the programmed group or the control group, and the adjusted mean EMA score for the programmed group was significantly higher than that of the control group. However, for both levels combined, only the comparisons involving the control group proved to be significant. In both cases the control group means proved to be lower than the other group with which it was compared.

On the basis of the criteria tests employed in this experiment, these findings indicate that when a distinction was made between the two levels it was found that neither material was more suitable for Level II. In fact, these students did not benefit from the enrichment program, whereas Level I students did benefit, the conventional material proving to be the more suitable for them. However, when the two levels were combined and considered as one, it appeared that the enrichment treatment groups did benefit from the enrichment program, but neither of the materials proved to be more suitable than the other.

Summary

Null Hypothesis I was accepted in its entirety, there being no significant differences among the three groups on the adjusted PMA test mean scores for either level separately or both levels combined. Null Hypothesis 2 was accepted for Level II, there being no significant differences among the three groups on the adjusted EMA test mean scores. However, the hypothesis was rejected for Level I and for both levels combined. For

Level I it was found that the adjusted EMA mean score for the conventional group was significantly higher than for either of the other groups, and that this mean for the programmed group was significantly higher than that of the control group. For both groups combined, the conventional group mean was not significantly higher than that of the programmed group, but the means of both these groups were significantly higher than that of the control group.

II. FINDINGS FROM THE TEACHER QUESTIONNAIRE

At the conclusion of the experiment in April, 1966, a questionnaire was completed by the 19 teachers involved. The purpose of the questionnaire was to determine the attitudes of the teachers toward the enrichment practices and procedures used in the experiment. Although the data collected did not lend itself to rigorous statistical analysis, it did provide important information concerning the success of the program. In this section, a summary of the information gathered from the teachers' responses to the questionnaire is presented in general terms with sample quotations of important teacher comments.

The Students

Five of the teachers were undecided about the practice of allowing Level I type students to engage in enriched mathematics study on their own. However, the remaining teachers felt that, as a result of this experiment, this type of student should be allowed to do so. Some of the reasons for the uncertainty on this point were made apparent by the following teacher comments:

> I do not feel able to judge at this point because I do not know what these students did in either of the tests.
>
> This depends on the individual students. Some, when working on their own will accomplish a great deal, others will do nothing.

These and other teacher comments indicated that the teachers' lack of knowledge of the criteria tests results, and the feeling that a decision on the point must be made on an individual basis for each student were the reasons for the indecision.

Only three teachers felt that Level II type students should be allowed to study mathematics enrichment materials on their own, and two felt that they should not. The remaining teachers indicated that they were undecided about this point. The same reasons for indecision on this point as in the case of Level I students was advanced by those teachers who offered a comment.

The Program

Six of the teachers were opposed, one undecided, and the remainder in favor of the practice of allowing students to use complete regular class mathematics periods once a week for mathematics enrichment. Some of the comments of those opposed were as follows:

> Not if they are expected to keep pace with the regular classes. However, if could be arranged that they do not miss an important lesson.

> Most timetables provide study periods for students. One of these a week could be used. The students could benefit from the enrichment program and still get a full week of regular classes.

One teacher who favored the practice provided a comment which would answer these criticisms satisfactorily in the investigators opinion.

> Teaching of the regular class could be adjusted to make certain the student does not miss new work.

All the teachers felt that the enrichment program did not interfere with their control of the students. However, eight of the teachers indicated that their teaching methods were interfered with to some extent. The fact that these teachers felt as they did on the latter point may not be a negative aspect of the experiment, for as one of them commented:

> I had to plan lessons (new) so that they would not interfere with the enrichment program.

The idea of a within-class enrichment program, as viewed by both the investigator and the administrators of the school system involved in the study, was that it should benefit the whole class, and not just the students engaged in the program. It was felt that this program should provide the opportunity for within-class grouping. During the enrichment periods, the average students might be working on practice exercises, the slower students could be receiving extra help from the teachers, and the

potentially talented students would be engaged in enrichment work on their own. One teacher who did not feel that his teaching methods had been disrupted was already practicing these ideas to some extent as was indicated by this comment:

> Students worked on the enrichment material in the second period of a double period, while other students worked on drill.

The majority of teachers thought that they should be allowed to exercise more influence over the students working on enrichment. Further questions revealed that the only type of increased influence they desired was that they be allowed to assist the students with content. Even then, there was some indication in their comments that this help should be minimal. Some teachers' comments on this point were as follows:

> Students should have some time where they could ask questions about the enrichment material.

> There should be assistance with the content but not to a large extent. Only when asked by the pupil.

It was only for the purpose of controlling the teacher effect for the experiment that the teachers were not allowed to assist the students with content.

Only one teacher felt that the enrichment program decreased the students' motivation in the regular class mathematics work. Two felt that the motivation was higher and the rest felt that it was about the same. Twelve teachers thought that their students were highly motivated by the enrichment, the students were motivated to some extent in the opinion of some teachers, and one felt that there was very little motivation. The only comment on this point was as follows:

> These students were very enthusiastic about working on their own. These students were, if anything, easier to control.

<u>The Material</u>

Fourteen of the teachers felt that the enrichment materials were suitable, and the remainder were undecided. One comment was that the students were bored with the programmed text. Another teacher stated that the materials did not seem to be related to the regular class mathematics content.

For Level I students, seven teachers thought that the programmed material was most suitable, five favored the conventional material, and the remainder were undecided. For Level II, nine preferred the programmed material, three the conventional material, and the rest were undecided. The following are two of the teachers' comments on this point:

> The higher the intelligence, the less programming that is required.

> When working on their own, students seem to need immediate knowledge of an answer.

General Comments

At the end of the questionnaire the teachers were asked to make any additional remarks about the program that they wanted to. Some of the important comments were as follows:

> I would be interested to know about the results of the entire group - assuming that the group consisted of varied abilities and ambitions - the results may be different from what I have observed in the students.

> With an enrichment program, I would like the students to be able to work on the program whenever they have the time rather than being restricted or have it set down for one period per week.

> The experiment tended to heighten the interest the students had in mathematics, but did not in many cases increase their mathematical achievement. If the student was not mathematically inclined it led to frustrations.

> The experiment worked successfully. It did not interfere with the work in class and, in fact, these students were able to grasp the ideas not taught to them very quickly. It probably should not be limited to class time but the students should be allowed to take it home if they so desire.

Two of these teachers expressed the opinion that the students should be allowed to work on the enrichment at other times, perhaps at home. This was not permitted in the experiment because it was desired to have all the students spend the same amount of time on the program. Furthermore, the influence of this program on other academic subjects was not measured. If the students worked on the enrichment at home or during study periods, it might adversely affect their achievement, in these other subjects. It was also felt that the kind of material used in this enrichment program required study by the students for periods of time comparable to normal class

period lengths to develop each major concept. Therefore, a few minutes here and there throughout the week whenever the students have time would not be satisfactory for this program.

Summary

Most teachers felt that Level I students should be able to work on mathematics enrichment on their own, but there was considerable uncertainty about Level II students. There was also some uncertainty about using regular class periods for the enrichment. Most teachers felt that the enrichment program did not interfere with their regular class procedures or the students' regular class mathematics work. They felt that the students were motivated by the enrichment, and that the programmed material was slightly more suitable. They felt that the content of both sets of material was suitable, but they would like to offer the students more assistance with it.

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS AND IMPLICATIONS

I. SUMMARY OF THE EXPERIMENT

Experimental Procedures

The purpose of the study was to test the suitability of a mathematics enrichment program which would be practical for the small junior high school and at the same time provide differentiated enrichment for different types of potentially talented mathematics students. The experimental group, consisting of 120 grade seven students with I.Q!s. of 115 or more, was categorized into two levels of 60 students each. Level I consisted of those students whose scores were among the top 60 on the PRMA test. This category represented those students who were high in both I.Q. and previous mathematics achievement. Level II consisted of those students whose scores were among the bottom 60 on the PRMA test. This category represented those students with high I.Q. but with previous mathematics achievement not commensurate with their ability. For each level separately, 20 students were randomly assigned to each of the two treatment groups, and to the control group.

There were two experimental treatments. One consisted of the study of a conventional mathematics enrichment text, Finite Mathematical Systems, and the other a programmed enrichment mathematics text, Modular Systems. The experimental groups studied their respective enrichment materials for one 30-minute regular mathematics period each week for eight weeks from February until April 1966. Meanwhile, the control group continued with regular mathematics classwork during these periods.

At the conclusion of the experiment, in April, the PMA, EMA and MAS tests were administered to all the students in the experiment. These tests were measures of the students' achievement in regular class mathematics taken up during the experimental period, achievement on the enrichment mathematics, and attitude toward mathematics respectively.

In addition, all the teachers of the students in the experiment completed a questionnaire on their attitudes toward the enrichment practices and procedures used in the experiment.

The null hypotheses to be tested were as follows:

1. With seventh grade students, there is no significant difference at the five percent level in regular mathematics classwork achievement among the control and experimental groups within
   a) Level I,
   b) Level II,
   c) Level I and II combined.
2. With seventh grade students, there is no significant difference in enrichment mathematics achievement among the control and experimental groups within
   a) Level I,
   b) Level II,
   c) Level I and II combined.

These hypotheses were tested by means of one-way analyses of covariance procedures. Control of the important variables of I.Q., previous mathematics achievement, and attitude toward mathematics was ensured experimentally by using these three variables as covariates with regular class mathematics achievement and enrichment mathematics achievement as each of the criteria in the two analyses. Other variables which might have affected the results were not controlled beyond randomly assigning the students to the experimental groups. However, it was found that the groups were reasonably comparable on these variables. In the statistical analysis of the data, the differences between adjusted mean scores were considered statistically significant if the probability of observing such a difference as a result of sampling error was .05 or less. The results of the responses to the teacher questionnaire were summarized in general terms.

Results of the Statistical Analyses

The statistical analyses carried out on the data in the experiment led to the following results:

There was no significant difference among the groups in regular mathematics classwork achievement for either Level I, Level II, or both levels combined.

There was no significant difference among the groups in enrichment mathematics achievement for Level II. However, for Level I and for both levels combined there were significant differences.

For Level I the adjusted mean on the EMA test for the conventional group was significantly higher than either that of the the programmed group or the control group. The mean for the programmed group was significantly higher than that of the control group.

For both levels combined there was no significant difference between the EMA group adjusted means of the conventional and the programmed groups, but both these means were significantly higher than that of the control group.

Findings from the Teachers Questionnaire

The teachers favored mathematics enrichment on their own for Level I students, but they were undecided about Level II, and about the practice of using regular class periods for this enrichment. The teachers felt that the program did not interfere with regular classroom procedures, nor with the achievement of the students engaged in the program. They felt that the students were motivated by the enrichment, and there was a slight preference among them for the programmed material. However, they indicated that the content of both materials was suitable for the enrichment program, but they also felt the need to offer the students some assistance with the content.

II. CONCLUSIONS

On the basis of the results of the statistical analysis and in consideration of the teacher responses to the questionnaire, the following conclusions were drawn:

For both previous mathematics achievement level categories separately and combined, there was no significant difference in achievement on regular classwork mathematics among the seventh grade students studying the two different types of mathematics enrichment materials, conventional and programmed

for regular 30-minute mathematics class periods once a week for eight weeks, and the students in the control group who were engaged in regular classwork mathematics during these periods. This finding indicated that regardless of previous mathematical achievement, students with an I.Q. of 115 or more could participate in some kind of mathematics enrichment program during one regular class period each week for a period of at least eight weeks without loss in regular classwork mathematics achievement. This view was also supported by most of the teachers involved in the experiment.

For students of the category designated as Level II, those whose mathematics achievement was not commensurate with their general ability, there was no significant difference in enrichment mathematics achievement among the three groups in the experiment. However, for Level I, those who were high in mathematics achievement as well as general ability, the conventional and the programmed group means were significantly higher than that of the control group, and the conventional group mean was significantly higher than that of the programmed group. With the exception that there was no significant difference in achivement in the enrichment mathematics between the conventional and programmed groups, the results for both levels combined was the same as for Level I. Although the number of teachers favoring the programmed material was slightly more than that of the teachers preferring the conventional material, about as many were undecided on this point.

These findings indicated that Level I type students could benefit from the type of enrichment program used in this experiment, and that the conventional material appeared to be the more suitable for them. Level II type students appeared to derive no benefit from this kind of enrichment program in terms of the mathematical outcomes which were measured in this experiment. The teachers also seemed to feel that the program would be unsuitable for these kinds of students. However, some of them felt this way because they thought that these students would suffer in their regular mathematics classwork. The results of this experiment indicate that this fear was unwarranted. Furthermore, many teachers recommended that they be allowed to assist the students with content. It may be that the absence

of this assistance was the reason for the failure of Level II students in this program.

The difference in the results for Level I and for the two levels combined indicated that if the students had been selected merely on the basis of I.Q., and not categorized on the basis of previous mathematics achievement, the conclusions drawn would have been quite different. It appears that the program was successful for all the students considered as a group, and that there was no difference in the suitabilities of the materials other than that the teachers were slightly in favor of the programmed materials. Therefore, it appears that catergorization of potentially talented mathematics students on the basis of previous mathematics achievement was a useful procedure.

III. LIMITATIONS

The foregoing conclusions must be considered in the light of the following limitations of the study:

These conclusions were limited in their interpretation to the population of seventh graders of the Edmonton Separate Schools from which the sample was drawn. Another possible limitation was the differences in the enrichment materials used. The different approaches of these materials may have led to bias in favor of one of the groups. However, the clock arithmetic approach of the conventional text was introduced into the the programmed text as an application. Likewsie, the remainder number approach of the programmed text was discussed to some extent in the conventional text.

The limitations of the instruments used to test mathematics achievement and attitude should be considered, since they were critical factors in determining the outcomes of the experiment. The investigator feels that the achievement tests did reflect the objectives of the authors of the regular curriculum and the enrichment texts, and also the objectives of the teachers, since most of them relied heavily on the textbook approach.

The fact that the I.Q., scores used in the selection of the population were taken from the tests administered to the students two years prior to the experiment may be a limiation. However, it was learned that the School

Board had no plans to alter their procedures for administering intelligence tests. Therefore, it was felt that it would be more practical to use the I.Q. data avilable because this is what would be used for any enrichment program that might be adopted.

A subjective examination of the data collected led to the decision that the assumptions underlying the analysis of covariance procedures used in this experiment were satisfied. It was felt that the limitations of this method of verifying the assumptions did not affect the outcomes of the experiment seriously because whenever observed differences among groups in the experiment led to the decision to reject the null hypotheses, the probabilities were very small. In other works, the chances were very small that the observed differences could results when the treatments applied were having no effect.

The fact that the attitude toward mathematics, which was used as a covariate, was measured after the treatments were applied might be a limitation of the experiment. In the literature it was indicated that enrichment experiences usually result in a more favourable attitude toward mathematics on the part of the students. However, in the experiments which produced this finding, the experimental periods were much longer than the eight week period of this experiment.

Finally, it is realized that the combining of the previous mathematics achievement levels for the purpose of testing the null hypotheses did not consititute satisfactory sampling procedures. However, it was felt that the importance of testing the usefulness of the categorizations, outwayed the disadvatages of the procedures.

IV, IMPLICATIONS FOR FURTHER RESEARCH

The experiment that has been reported here should be replicated with more rigorous controls. In order to conduct more complex experimentation such as two-way analyses, more treatment groups, or more categories it would be necessary to use a larger sample. In order to generalize the conclusions to a larger population, random samples should be drawn from different populations.

It would also be desirable to use enrichment materials designed specifically for the purpose of the experiment, so that the exact differences and similarities in the materials could be rigorously controlled. The experiment should be conducted over longer periods than used in this experiment.

The testing of other kinds of enrichment materials or experiences for potentially talented mathematical students which would be practical for small junior high schools should be the subject of further investigations. There seems to be a particular need for this in the case of Level II type achievers. In fact, for this type of student, it might be useful to compare other kinds of special programs, such as remedial work, with enrichment programs.

If some means of control of the teachers' influence factor can be found, an experiment in which teachers are allowed to offer more assistance to the student, particularly in regard to content, should be undertaken.

Different categorizations of potentially talented students could serve as the basis of further investigations.

In addition to working on enrichment in the regular class time, students should be allowed to work on the enrichment at other times in school or at home when they have the time and feel motivated to do so. However, the effect of this extra enrichment time on the other academic subjects should be determined experimentally.

Finally, if important outcomes of mathematics learning other than those measured in this experiment, can be identified and measured, enrichment experiments designed to measure these outcomes should be undertaken.

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APPENDIX A

RAW SCORES FOR ALL STUDENTS

The scores obtained by the students which constituted the data of the statistical analysis are presented on the following pages.

The symbols heading the columns are to be interpreted as follows:

ID (Column 1): Identification number
IQ (Column 2): Intelligence Quotient Score
PRMA (Column 3): Pre-Mathematics Achievement Score
MAS (Column 4): Mathematics Attitude Score
PMA (Column 5): Post Mathematics Achievement Score
EMA (Column 6): Enrichment Mathematics Achievement Score

ID	IQ	PRMA	MAS	PMA	EMA
1101	128	40	77	40	19.5
1102	132	38	98	35	19.5
1103	138	36	110	23	16
1104	123	39	73	35	14
1105	120	38	69	34	17
1106	119	36	109	37	17
1107	112	35.5	84	34	15
1108	120	35.5	92	36	17
1109	131	38.5	91	31	16
1110	129	36	83	28	16
1111	125	38	103	23	17
1112	133	37.5	69	19	18
1113	120	37	99	25	12
1114	125	37.5	81	26	17
1115	125	37	70	28	17.5
1116	129	36.5	99	18	13
1117	125	36	96	27	15
1118	130	36	97	33	15
1119	136	36	79	31	18
1120	132	35.5	56	28	11
1201	131	37	90	21	15
1203	115	37	96	35	11
1204	128	36	44	31	14
1205	115	36	95	33	17
1206	132	38.5	83	34	16
1207	116	39	97	27	14
1208	128	36.5	90	21	14
1209	116	36	93	23	12
1210	118	36	85	32	16
1211	124	39	100	38	16
1212	119	37.5	83	24	13
1213	119	37	71	17	14

ID	IQ	PRMA	MAS	PMA	EMA
1214	130	38	95	37	17
1215	125	36.5	96	17	13.5
1216	116	38	80	25	14
1217	133	37	88	36	15
1218	134	35.5	85	20	14
1219	136	37	78	39	15
1220	122	37	64	26	12
1301	130	39	90	37	15
1302	124	39	90	38	16
1303	115	37	75	35	12
1304	137	36	85	38	13
1305	134	36	87	37	13
1306	118	35.5	64	35	15
1307	130	37	85	30	13
1308	122	35.5	91	22	8
1309	134	35.5	89	21	5
1310	125	38.5	93	35	11
1311	134	37	90	27	14
1312	126	36	74	17	6
1313	124	36	85	31	9
1314	123	37	71	36	12
1315	139	36.5	77	35	15
1316	116	36	89	27	11
1317	142	39	81	24	11
1318	119	36	75	20	15
1319	120	36	74	22	11
1320	114	36	70	25	7
2101	117	28	97	22	14
2102	115	26	81	15.5	13
2103	123	28.5	74	23	11
2104	115	26	76	17	11
2105	118	27	88	16	10
2106	117	26.5	76	16	12
2107	115	26	69	12	9
2108	121	25	75	16	10
2109	128	28.5	60	21	9
2110	117	27	92	18	11
2111	121	25.5	62	14	4
2112	115	27	80	16	14
2113	118	28	66	18	9
2114	125	25.5	75	16	14.5
2115	120	28.5	94	14	10
2116	120	24	96	14	7
2117	124	28	70	18	12
2118	129	28	67	12	12.5
2119	114	27	59	26	10
2120	114	26	71	18.5	10
2201	112	27	72	24	14

ID	IQ	PRMA	MAS	PMA	EMA
2202	114	25.5	95	16	12
2203	117	23.5	67	10	7
2204	117	27	66	13	8
2205	115	28	105	13	13
2206	117	23	77	25	12
2207	115	23	62	16	6
2208	118	27	74	19	12
2209	114	23.5	71	19	7
2210	108	28	71	14.5	12
2211	116	28	54	19	13
2212	115	23	91	8	12
2213	117	27	80	21	8
2214	115	24.5	81	10	7
2215	116	27	89	19	13
2216	123	27.5	84	15	13
2217	128	25	67	18	7
2218	115	20	51	9	10
2219	115	24.5	67	36	11
2220	118	28.5	93	18	11
2301	118	28.5	80	22	9
2302	115	24	48	18	10
2303	130	28	80	24	9
2304	120	23	99	27	10
2305	120	26	79	14	8
2306	124	26	61	16	11
2307	119	27	54	15	14
2308	118	23.5	86	14	5
2309	124	26	75	24	15
2310	120	26	90	24	12
2311	120	25	73	22	12
2312	128	28	72	22	10
2313	121	26	70	22	11
2314	118	27	80	16	7
2315	115	27	68	18	12
2316	118	25	62	17	9
2317	122	26.5	76	22	11
2318	117	28.5	77	18	14
2319	118	27	74	27	8
2320	121	26	76	23	8

ID No's: 1101 - 1120 inclusive — Level I, conventional
1201 - 1220 inclusive — Level I, programmed
1301 - 1320 inclusive — Level I, control
2101 - 2120 inclusive — Level II, conventional
2201 - 2220 inclusive — Level II, programmed
2301 - 2320 inclusive — Level II, control

APPENDIX B

FINAL VERSION

PRE - MATHEMATICS ACHIEVEMENT TEST

I. For each of the following exercises select the best answer to each question and place the letter of that answer above the line to the right of the question.

1. $4 \times 2 + 3 =$ a. 20 b. 14 c. 24 d. 11 ______

2. $5 \times (3 + 2) =$ a. 17 b. 10 c. 25 d. 30 ______

3. 1,052,052,703 is read

   a. One thousand and fifty two billion seven hundred and three

   b. one billion fifty two million fifty two thousand seven hundred and three

   c. one thousand and fifty two million seven hundred and three

   d. one thousand and fifty two billion fifty two thousand seven hundred and three ______

4. $2 \div 6 =$ a. $6 \div 2$ c. 3

   b. $1 - 3$ d. 12 ______

5. 2 and 3 are factors of

   a. 2 b. 5 c. 15 d. 12 ______

6. IV a. 4 b. 5 c. 6 d. 9 ______

7. $5 + (3 + 1) =$ a. $4 + 5$ c. $5 + 3$

   b. $15 + 1$ d. $5 \times (3 + 1)$ ______

8. Which of the following pairs of sets are equivalent? (U=N)

   a. $\{1,2,3,\ldots\}$; $\{2,1,3,\ldots\}$ c. $\{1,2,3,4\}$; $\{4,3,5,2,1\}$

   b. $\{2,3,4\}$; $\{2,3,4,\ldots\}$ d. $\{1,2,3,\ldots\}$; $\{1,2,3\}$

   ______

9. $1 \times n =$ a. 0 b. n c. $n + 1$ d. 1 ______

10. $9 \times 4 =$ a. $4 \times 4 \times 4 \times 4$ c. $9 + 9 + 9 + 9$

    b. ~~4xxx3~~ 9^4 d. $9 \times 9 \times 9 \times 9$ ______

11. $4^3 =$ a. 3^4 c. $4 \times 4 \times 4$

    b. 4×3 d. $3 \times 3 \times 3 \times 3$ ______

12. Each member of the set $\{1,2,3,5,7,11,13\}$ has the property that

a. all are odd numbers
b. None are divisible by 1
c. all are divisible by zero
d. none are divisible by any natural number not in the set. ______

13. If c x a/b = 1, then c =

a. a/b   b. a   c. b/a   d. b ______

II. In each of the exercises below a number of responses are listed. For each part of each exercise, select from these the best response. You may use each response once, more than once, or not at all. Write the letter(s) of the best response(s) in the spaces within each part of each exercise.

A. a. solution set   c. universe   e. member
b. subset   d. tabulation

14. $\{0,1,2\}$ is a ______ of $\{x | x < 5\}$. U=N.
15. Two is a ______ of $\{x\ x < 5\}$. U=N.
16. $\{3\}$ is the ______ of x + 2 = 5. U=N.
17. $\{7,8,9\}$ is a ______ of $\{x | x < 10\}$. U=N.
18. $\{1,3,5,7\}$ is a ______ of a subset of N.

B. Select the tabulation(s) of the solution sets for each condition. U=N.

a. $\{1192\}$ c. $\{160\}$ e. $\{122\}$
b. $\{540\}$ d. $\{326\}$

19. 535 + x = 657 ____   22. a - 433 = 759 ____
20. y - 535 = 657 ____   23. t - 107 = 433 ____
21. 243 - n = 83 ____   24. 350 + 190 = d ____

C. Select from the list the sentence(s) that express(es) a condition for the problem.

a. x - 23 = 10   c. 23 + 10 = x   e. 23 + 10 > x
b. x + 10 = 23   d. x - 23 > 10   f. x - 10 < 23

22. Bill had 23 model airplanes. After he bought 10 more model airplanes, he had as many models as Jim. How many models does Jim have. ______

D.

26. The solution set of which condition expressed below is an infinite set? U=N.

a. $294 - n > 5$ c. $n - 7 = 83$

b. $45 + n < 76$ d. $n + 3 > 7$

27. What name do we give to the set that has no members?

a. Finite set c. Empty set

b. Infinite set d. Subset

E. Select from the responses the tabulation(s) of the sets named in each part of the exercise.

a. $\{1,2,3,4,5\}$ c. $\{2,3,4\}$ e. $\{\ \}$

b. $\{2,3\}$ d. $\{3\}$

28. The intersection of $\{1,2,3\}$ and $\{2,3,4,5\}$.

29. The union of $\{1,2,3\}$ and $\{2,3,4,5\}$.

30. $\{3,4,5,\ldots\} \cap \{0,1,2,3\}$. U=N.

31. $\{2,3,4\} \cup \{\ \}$.

32. $\{2,3,4\} \cap \{\ \}$.

33. $\{2,3,4\} \cup \{2,3,4\}$.

F.

34. Suppose that the union of sets R and S is the empty set. What do you know about sets R and S?

a. Sets R and S meet.
b. R contains at least one member that is not in S.
c. Each set is the empty set.
d. S contains at least one member that is not in R.

35. What is the solution of the following problem?

The sum of Tom's and Jack's ages is 18. In four years Tom will be as old as Jack is now. How old is each boy now? U = C X C.

a. Tom is 7 years old, and Jack is 11 tears old.
b. Tom is 14 years old, and Jack is 18 years old.
c. Tom is 4 years old, and Jack is 14 years old.

36. Select the correct condition(s) for the following problem:

The sum of two numbers is 7. The result of subtracting the second number from the first is 5. What are the two numbers?

a. $x + y = 7$ $x - y = 5$ b. $x + y = 7$ c. $x + y = 7$ $5 - y = x$

III. If a sentence expresses a true statement write T, if false write F.

37. If A = (1,2) , then $\{(1,1),(2,1),(1,2),(2,2)\}$ is a tabulation of the Cartesian set A X A.

38. The set $\{(2,3)\}$ has exactly one member.

39. If U = N X N, there is exactly one point in the graph of $\{(x,y) \mid x = 2\}$

40. (5,10) is the same ordered pair as (10,5).

EDMONTON SEPARATE SCHOOLS

GRADE SEVEN POST-EXPERIMENT MATHEMATICS TEST

DATE______________ NAME ______________

Part I - Thirty Minutes SCHOOL ______________

1. There are two common ways in which modular addition can be represented. For instance, for 1 plus 4 equals 0 in mod 5, we may write $[1]_5 + [4]_5 = [0]_5$ or we may write $1 + 4 = 0$ mod 5.

   i. Which statement is equivalent to $[4]_6 + [1]_6 = [5]_6$?

   a. $6 + 4 = 10$ mod 5 b. $4 + 1 = 5$ mod 6

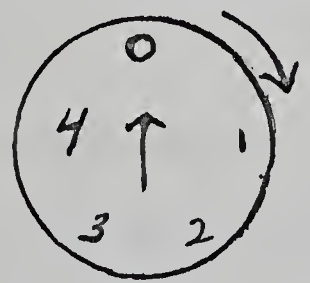
   c. $1 + 4 = 6$ mod 5 d. $4 + 1 = 6$ mod 5 ______

   ii. Which statement is equivalent to $5 + 2 = 0$ mod 7 ?

   a. $[5]_0 + [2]_0 = [7]_0$ b. $[5]_7 + [2]_7 = [0]_7$

   c. $[0]_5 + [7]_5 = [2]_5$ d. $[7]_5 + [7]_2 = [7]_0$ ______

2. Using the clock below for modular addition, counting clockwise, in which modular system should you do the addition ?



   a. Mod 0 b. Mod 3

   c. Mod 4 d. Mod 5 ______

3. The operation represented by $\oplus$ is called addiv. To find the addiv of two numbers, find the sum of the two numbers and divide by the modular number. The remainder is the addiv of the two numbers. For example, in the mod 5 system, $4 \oplus 3 = 2$ because the sum of 4 and 3 is 7, and 7 divided by the modular number 5 is 1 with a remainder of 2.

   i. What is $4 \oplus 4$ in mod 5 ?

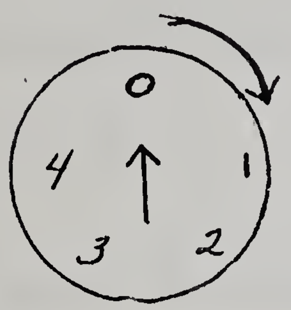
   a. 8 b. 1 c. 0 d. 3 ______

3. continued

ii. What is $4 \oplus 4$ in mod 8 ?

a. 8 b. 1 c. 0 d. 3 ______

4. Using the clock below to represent the modular system, and counting clockwise, which of the following is a true statement ?



a. 2 + 4 = 1 mod 4

b. 3 + 2 = 0 mod 5

c. 2 + 0 = 2 mod 4

d. 4 + 5 = 4 mod 5 ______

5. How many multiplications (of two numbers) are possible in the mod 6 system ?

a. 12 b. 2 c. 36 d. 6 ______

6. In the mod 7 system, which of the following statements are true ?

a. 6 + 7 = 7 + 6 b. 2 + 2 = 0

c. 3 + (2 + 6) = (3 + 2) + 6 d. 4 + 4 = 8 ______

7. If $8 \circledvee 2$ means the average of 8 and 2, the answer is 5 because 8 plus 2 is 10, and 10 divided by 2 equals 5, which is the average of 8 and 2. Does the associative property apply to $8 \circledvee (2 \circledvee 2) = (8 \circledvee 2) \circledvee 2$?

a. YES b. NO ______

8. Which of the following is an example of the associative property and a true statement in mod 9 arithmetic ?

a. 9 + (8 + 7) = (9 + 8) + 7 b. (6 + 6) + 8 = 6 + (6 + 8)

c. 6 + 7 = 7 + 6 c. 7 x (8 + 2) = (7 x 8) + (7 x 2) ______

9. What is the identity element for addition in the mod 11 system ?

a. 0 b. 11 c. 1 d. 2 ______.

10. What is the additive inverse of zero in the mod 2 system ?

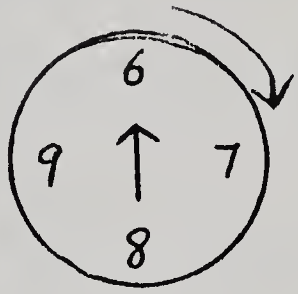
a. 0 b. 11 c. 1 d. 2 ______

11. What is 1 plus 1 in mod 2 ?

a. 0 b. 11 c. 1 d. 2 ______

12. Using the following addition table

+	6	7	8	9
6	8	9	6	7
7	9	6	7	8
8	6	7	8	9
9	7	8	9	6



i. Is this system closed for addition? a. YES b. NO ______

ii. Is the system commutative? a. YES b. NO ______

iii. Is the system associative? a. YES b. NO ______

13. Two dollars worth of gas lasts Martin five days. On one Monday morning Martin bought twice his usual amount of gas. Assuming he continues to use gas at his usual rate, and that he does not fill his tank again until the level of gas in his tank is down to what it usually is when he goes to get more,

i. On what day of the week will he go to buy more gas ?

a. Wednesday b. Thursday c. Friday d. Saturday ______

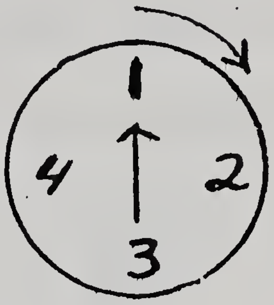
ii. Which modular system should you use to calculate the answer to the problem?

a. Mod 7 b. Mod 5 c. Mod 2 d. Mod 10 ______

(4.)

14. The operation R is defined as moving from one room to another in the following order: living room (1), dining room (2), kitchen (3), bedroom (4). For example, three rooms from the dining room (2) is the living room (1). The R table is as follows:

R	1	2	3	4
1	2	3	4	1
2	3	4	1	2
3	4	1	2	3
4	1	2	3	4



i. Is the operation R commutative ? a. YES b. NO ______

ii. What is the identity for the operation R ?

a. 1 b. 2 c. 3 d. 4 ______

iii. What is the inverse of 2 for the operation ?

a. 1 b. 2 c. 3 d. 4 ______

15. In ordinary arithmetic $\square \times (\triangle + \nabla) = (\square \times \triangle) + (\square \times \nabla)$. We call this the distributive property of multiplication. Does modular multiplication have the property of distributivity ?

a. YES b. NO ______

PART II - Thirty minutes. This part may be written separately from the first part, but remember, you must spend only thirty minutes on each part.

1. Five sets are tabulated below. For each exercise, select from the list the tabulation of the set described. You may use a response once, more than once, or not at all. The universe for each variable is C.

a. $\{45\}$ b. $\{5\}$ c. $\{36\}$ d. $\{60\}$ e. $\{55\}$

i. $y/6 \sim 42/7$ ______ iv. r is 15% of 400 ______

ii. $65/91 \sim p/7$ ______ v. $45/117 \sim s/13$ ______

iii. $13/117 \sim 5/x$ ______ vi. 11 is 20% of t ______

2. Five rate pairs are expressed below. For each exercise, select from this list a rate pair that is a member of the solution set of the condition. The universe for (x,y) is N X N.

a. 25/60 b. 6/10 c. 27/15 d. 3/1 e. 7/13

i. $x/y \sim 9/5 \wedge x - y = 12$ ______ iv. $5/12 \sim y/x \wedge x > y + 34$ ______

ii. $y/x \sim 14/26 \wedge y + 6 = x$ ______ v. $x/y \sim 54/30 \wedge x - 12 = y$ ______

iii. $12/4 \sim x/y \wedge x - 2 = y$ ______ vi. $x/y \sim 3/5 \wedge x + y = 16$ ______

3. For each problem below, select from the list above the problems, a sentence that expresses a condition for the problem. You may use a response once, more than once, or not at all. U = C X C.

a. $40/x \sim 300/100 \wedge y - x = 300$ b. $x/y \sim 40/100 \wedge y + 300 = x$

c. $x/300 \sim 40/100 \wedge x + y = 300$ d. $x/y \sim 40/100 \wedge x - y = 300$

e. $x/300 \sim 30/100 \wedge 40 + y = x$ f. $x/y \sim 40/100 \wedge y - x = 300$

3. continued

i. Jack wants to buy a scooter which sells for $300. He has already saved 40% of the amount he needs. Which condition should you use to find how many more dollars he must save to buy the scooter? ______

ii. A certain rate pair is equivalent to 40/100 and has a second component that is 300 more than the first component. Which condition should you use to find the rate pair? ______

4. Five numerals are listed below. For each exercise, select from the list the decimal numeral that expresses the number named. Each numeral listed as a response may be used once, more than once, or not at all.

a. 65 b. 19 c. 59 d. 160 e. 9

i. 1001_{two} ______ iii. 2012_{three} ______ v. 316_{seven} ______

ii. 34_{five} ______ iv. 101_{eight} ______ vi. 114_{twelve} ______

5. For each of the following exercises, four responses are listed below the exercise. Select the response that best answers the question in the exercise or that correctly completes the exercise.

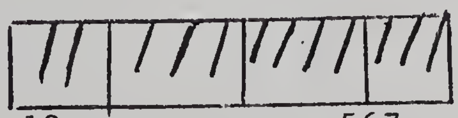
i. The exponent of 5^4 is ______

a. 5 b. 4 c. 1 d. 20

ii. Nine is a factor of ______

a. 3 b. 9 c. 29 d. 39

iii. What number does the base - six numeral below express ?



a. 2343 b. 12 c. 567 d. 564 ______

6. An astronaut who landed on a planet in another solar system found that the inhabitants used a base four system of numeration with the property of place value. When the inhabitants counted they used the words ul, nub, lum, ul tu, ul ul, and so on.

i. What decimal numeral would express the number ul ul ?

a. 4 b. 10 c. 1 d. 5 ______

ii. What is the name used by these inhabitants to express the base four numeral 13 ?

a. ul ul tu b. ul lum c. ul ul ul d. tu lum ______

iii. What is the name used by these inhabitants to express our decimal numeral 7 ?

a. ul lum b. ul ul ul c. ul ul tu d. tu lum ______

7. For each of the following exercises, four responses are listed below the exercise. Select the response that best answers the question in the exercise or that correctly completes the exercise.

i. In the base-four grouping system that does not have place value, there must be a special symbol for the number ______

a. 2 b. 8 c. 10 d. 16

ii. $3(10^3) + 5(10^2) + 0(10^1) + 2$ is the expanded form of ______

a. 12 b. 352 c. 3502 d. 3352012

iii. Which digit of the numeral 6407 is associated with the second power of the base ? ______

a. 6 b. 0 c. 4 d. 7

(4.)

7. continued

iv. Which numeral expresses the number one hundred million two hundred sixty-five thousand thirty ? ______

a. 100,200,065,030 b. 100,260,005,030

c. 100,260,530 d. 100,265,030

v. The base-six numeral that expresses the sum of 42_{six} and 35_{six} is

a. 77_{six} b. 121_{six} c. 115_{six} d. 1111_{six} ______

vi. In the numeral 351,135,413,315, which digits form the millions period?

a. 351 b. 135 c. 413 d. 315 ______

vii. If 2 X 6 = 10 expresses a true statement, the numerals 2, 6, and 10 must be written in base ______.

a. six b. five c. ten d. twelve

viii. What is the base of the place value numeration system that uses exactly nine different digits ? ______

a. ten b. nine c. eight d. seven

ix. The number five million is expressed in scientific notation as

a. 5×10^6 b. 5×10^5 c. 5×10^7 d. 5×10^{10} ______

x. If the sentence 1 + 1 = 10 expresses a true statement, the numerals 1, 1, and 10 are written in base ______

a. zero b. one c. two d. ten

xi. The digit 3 in the numeral 2357_{seven} expresses ______.

a. 21 b. 147 c. 300 d. 2100

7. continued

xii. The numeral that expresses the number one thousand nine hundred forty-six is ______.

a. MCMXLVI b. MCMLXIV c. MDCDXLVI d. MDCCCCLXVI

xiii. The numeral $\mathcal{X}2\mathcal{E}_{twelve}$ expresses the same number as the base-ten numeral ______.

a. 1596 b. 1475 c. 133 d. 23

xiv. 770_{eight} is how many times as great as 77_{eight}? ______

a. two b. four c. eight d. ten

EDMONTON SEPARATE SCHOOLS

A MATHEMATICS STUDY

April, 1966

NAME OF SCHOOL ______________________

The best answer to each statement is your own first impression. There are no right or wrong answers. Think carefully, but do not spend too much time on any one question. Let your own personal experience guide you to choose the answer you feel about each statement.

Please mark a response for every statement.

ANSWERS

1. I find most mathematics lessons:
   a) extremely interesting.
   b) quite interesting.
   c) interesting
   d) not very interesting.
   e) not interesting at all. ______

2. A knowledge of mathematics for any job at all is:
   a) most important.
   b) very important
   c) quite important.
   d) of small importance.
   e) not important. ______

3. If I did not have to take mathematics, I would like school:
   a) much less.
   b) a little less.
   c) same as now.
   d) a little better
   e) much better. ______

4. Mathematics is:
   a) the most important subject.
   b) one of the more important subjects.
   c) just as important as any other subject.
   d) not as important as some of the other subjects.
   e) the least important subject. ______

5. I find problem solving:
   a) extremely interesting.
   b) quite interesting.
   c) interesting.
   d) not very interesting.
   e) not interesting at all. ______

6. When I have difficulty with a new topic in my mathematics course, I ask my teacher to clarify the section:
   a) very frequently.
   b) frequently.
   c) sometimes.
   d) hardly ever.
   e) never. ______

7. If books about mathematics are available, I would:
   a) read most of them.
   b) read some of them.
   c) look at the diagrams and pictures.
   d) page through some of them.
   e) never look at them. ______

8. If someone says mathematics classes are worthless and a waste of time, I would:
   a) strongly disagree.
   b) tend to disagree.
   c) not take a side.
   d) tend to agree.
   e) strongly agree. ______

9. When I do my homework, my mathematics is:
   a) always done first.
   b) often done first.
   c) usually done first.
   d) sometimes done first.
   e) never done first. ______

10. I find mathematical puzzles:
   a) extremely interesting.
   b) quite interesting.
   c) sometimes interesting.
   d) not very interesting.
   e) not interesting at all. ______

11. I would be interested in taking other subjects that make use of:
   a) a great deal of mathematics.
   b) quite a bit of mathematics.
   c) some mathematics.
   d) a little mathematics.
   e) no mathematics. ______

12. If given the opportunity to join one of the following clubs, I would prefer a:
    a) mathematics club.
    b) science club (physics).
    c) science club (chemistry).
    d) science club (geology).
    e) literary club. _____

13. If I could receive one of the following magazines for a year, I would pick:
    a) a mathematics magazine for high school students.
    b) a magazine combining science and mathematics for high school students.
    c) a science magazine for high school students.
    d) a geology magazine for high school students.
    e) a literary magazine for high school students. _____

14. When I study my mathematics course, I most often:
    a) make written summaries of the sections covered.
    b) do additional problem solving.
    c) do many drill questions.
    d) memorize the formulas given in the text.
    e) look over some work done previously. _____

15. If I listed my courses in order preference, I would place mathematics:
    a) first.
    b) second.
    c) third.
    d) fourth.
    e) fifth. _____

16. Whenever mathematical problems are presented to us for solving, I get:
    a) a great deal of satisfaction in working them out.
    b) quite a bit of satisfaction in working them out.
    c) some satisfaction in working them out.
    d) very little satisfaction in working them out.
    e) no satisfaction in working them out. _____

17. My mathematics course has made:
    a) mathematics enjoyable for me.
    b) mathematics a pleasant course.
    c) me feel indifferent towards mathematics.
    d) mathematics classes an uncomfortable experience for me.
    e) me strongly dislike mathematics. _____

18. When I do my mathematics homework, I am usually:
    a) extremely interested.
    b) interested.
    c) somewhat interested.
    d) not too interested.
    e) not interested at all. ____

19. When we start a new topic in mathematics, I am usually:
    a) keenly interested.
    b) interested.
    c) somewhat interested.
    d) not too interested.
    e) not interested at all. ____

20. The average amount of time I spend on homework assignment in mathematics takes the following time per day:
    a) more than one hour.
    b) 3/4 hour to one hour.
    c) 1/2 hour to 3/4 hour.
    d) 1/4 hour to 1/2 hour.
    e) 0 hours to 1/4 hour. ____

21. When I get an assignment in mathematics:
    a) I do it immediately.
    b) I do it eventually.
    c) I may get it done.
    d) I put it off as long as possible.
    e) I don't do it. ____

22. Most of my work in this class is done:
    a) to satisfy my curiosity about mathematics.
    b) to gain competence in mathematics.
    c) to get a good mark.
    d) to just pass the class.
    e) to put in the time alloted to mathematics. ____

23. During mathematics lessons, I feel:
    a) extremely confident in myself.
    b) quite confident in myself.
    c) confident in myself.
    d) a little unsure of myself.
    e) very unsure of myself. ____

APPENDIX C

ITEM DISCRIMINATION AND DIFFICULTY INDICES FOR PILOT STUDY VERSIONS OF PRMA AND PMA TESTS AND FINAL VERSION OF EMA TEST

PMA TEST						PRMA TEST						EMA TEST		
n	r	p	n	r	p	n	r	p	n	r	p	n	r	p
38	.92	.49	4	.38	.64	4	.89	14	7	.41	.43	14	.82	.89
30	.833	.72	29	.37	.58	35	.72	58	36	.39	.57	5	.77	.33
27	.81	.27	40	.37	.17	11	.69	48	6	.39	.82	11	.77	.78
14	.74	.47	26	.35	.11	10	.65	77	23	.39	.41	3	.71	.89
10	.63	.32	21	.35	.89	40	.65	54	21	.38	.31	15	.70	.85
24	.63	.18	6	.35	.55	5	.61	30	32	.38	.30	17	.70	.12
13	.63	.19	15	.34	.62	9	.60	49	17	.37	.52	18	.68	.26
35	.55	.69	8	.33	.57	39	.58	61	31	.37	.61	8	.64	.61
9	.54	.53	72	.32	.57	34	.58	55	18	.35	.25	6	.59	.15
28	.51	.46	31	.32	.28	38	.58	79	30	.35	.28	9	.59	.22
3	.51	.42	12	.32	.60	26	.55	63	22	.35	.39	13	.59	.88
36	.51	.73	34	.31	.56	33	.51	78	15	.34	.41	4	.53	.21
1	.51	.82	33	.31	.14	32	.51	45	3	.34	.87	19	.51	.27
37	.47	.12	25	.30	.13	25	.48	45	28	.33	.81	12	.47	.73
2	.46	.70	20	.30	.42	1	.45	75	8	.33	.15	20	.38	.12
11	.43	.43	16	.30	.40	20	.45	73	29	.33	.59		.33	.89
23	.43	.32	19	.28	.19	13	.44	26	14	.33	.72	10	.33	.67
39	.39	.47	18	.27	.21	24	.44	78	16	.32	.54	7	.33	.21
7	.38	.56	17	.25	.48	12	.43	35	37	.29	.47	16	.33	.33
32	.38	.83	5	.24	.68	19	.43	43	27	.27	.61	2	.33	.77

ITEMS DISCARDED

PMA r	PMA p	PMA r	PMA p	PRMA r	PRMA p	PRMA r	PRMA p	EMA			
.10	.35	.13	.69	N	.12	.30	.89	N	.82	.21	.82
.23	.23	.22	.14	.21	.23	N	.17	N	.57	N	.79
N	.31	.21	.43	N	.35	N	.14	N	.72	.30	.21
N	.17	.22	.22	.12	.25	.27	.14	N	.20	.33	.10
.18	.67	N	.31	N	.81	N	.78	N	.43	N	.47

n = item number   r = tetrachoric correlation (index of discrimination)
p = proportion of students in pilot study who passed item (index of difficulty)

[illegible]

[illegible]

[illegible]

DATA FOR CALCULATION OF RELIABILITY INDICES FOR FINAL VERSIONS OF PRMA, PMA, AND EMA TESTS

PRMA TEST				PMA TEST				EMA TEST		
n	p	n	p	n	p	n	p	n	p	
1	.88	21	.57	1	.29	21	.34	1	.89	
2	.55	22	.55	2	.94	22	.79	2	.77	
3	.94	23	.50	3	.97	23	.69	3	.89	
4	.35	24	.98	4	.54	24	.92	4	.21	
5	.59	25	.49	5	.79	25	.85	5	.33	
6	.95	26	.69	6	.96	26	.92	6	.15	
7	.63	27	.64	7	.82	27	.88	7	.21	
8	.49	28	.89	8	.59	28	.97	8	.61	
9	.58	29	.71	9	.89	29	.83	9	.22	
10	.98	30	.29	10	.96	30	.36	10	.67	
11	.62	31	.59	11	.62	31	.88	11	.78	
12	.51	32	.57	12	.45	32	.82	12	.73	
13	.39	33	.79	13	.78	33	.97	13	.88	
14	.88	34	.65	14	.93	34	.79	14	.89	
15	.39	35	.76	15	.73	35	.91	15	.85	
16	.62	36	.67	16	.90	36	.77	16	.33	
17	.56	37	.59	17	.72	37	.78	17	.12	
18	.59	38	.92	18	.94	38	.98	18	.26	
19	.55	39	.84	19	.78	39	.89	19	.27	
20	.77	40	.59	20	.89	40	.74	20	.12	
40				40				20		K
31.28				65.38				10.33		S_x^2
7.77				7.87				3.25		P (1-P)
.77				.90				.72		r.i.

r.i.= RELIABILITY INDEX $= \left(\frac{k}{k-1}\right)\left(1 - \frac{\sum_{g=1}^{k} P(1-P)}{S_x^2}\right)$

r.i. = Reliability Index

K = Number of Items

S_x^2 = Test Score Variance

P = Proportion of students on test final version who passed the item

n = item number

[illegible]

[illegible]
[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

CALCULATION OF SIZE OF SAMPLE

TOTAL SCORE	=	100%
DIFFERENCE IN SCORE BETWEEN GROUPS CONSIDERED SIGNIFICANT, $(\mu_i - \mu)$	=	5%
NUMBER OF TREATMENT GROUPS, K	=	3
VARIANCE ON PRMA SCORES, σ^2	=	79.21
LEVEL OF SIGNIFICANCE, α_x	=	.05
POWER	=	.90

$$\phi' = \sqrt{\frac{\Sigma(\mu_i - \mu)^2 / k}{\sigma_x^2}} = \sqrt{\frac{(5^2 + 5^2 + 5^2)/3}{79.21^2}} = .56$$

REFERRING TO TABLE B II (WINER, P. 657):

<u>n = 17</u>

COMPUTATION FORMULAS

One Way Analysis of Covariance with Three Covariates

Analysis of Variance and Analysis of Covariance

SOURCE	SS	df	MS	F
GROUP	T_{yy}	k - 1	MS_{group}	$\frac{MS_{group}}{MS_{error}}$
ERROR	E_{yy}	k (n -1)	MS_{error}	
TOTAL	S_{yy}	k n - 1		
GROUP	T'_{yy}	k - 1	MS'_{group}	$\frac{MS'_{group}}{MS'_{error}}$
ERROR	E'_{yy}	k (n - 1)-2	MS'_{error}	
TOTAL	S'_{yy}	kn - 3		

Computation Formulas

Covariate Variance Terms, i = 1,2,3	Covariance Terms i = 1,2,3	Criterion Variance Terms
$(1x_i) = G_{x_i}^2/kn$ $(2x_i) = \sum x_i^2$ $(3x_i) = \sum T x_{ij}^2/n$	$(1x_iy) = G_{x_i} G_y/kn$ $(2x_iy) = \sum x_i y$ $(3x_iy) = (\sum T x_{ij} T_{yj})/n$	$(1y) = G_y^2/kn$ $(2y) = \sum y^2$ $(3y) = (\sum T_{yj}^2)/n$

G_{x_i} - the sum of all covariate measures
G_y - the sum of all criterion measures

Anovo Formulas

Covariates	Criterion
$S_{xx} = (2x_i) - (1x_i)$ $E_{xx} = (2x_i) - (3x_i)$ $T_{xx} = (3x_i) - (1x_i)$	$S_{yy} = (2y) - (1y)$ $E_{yy} = (2y) - (3y)$ $T_{yy} = (3y) - (1y)$

Ancova Formulas

$$T'_{yy} = S'_{yy} - E'_{yy}$$

$$E'_{yy} = E_{yy} - \sum_{i=1}^{3} b''_{y \cdot x_i} E_{x_i y}$$

$$S'_{yy} = S_{yy} - \sum b_{y \cdot x_i} S_{x_i y}$$

Hartly F - max Test

$$F_{max} = \frac{S^2 \text{ largest}}{S^2 \text{ smallest}} \qquad \text{where} \qquad S^2 = \frac{\sum X_j^2 - T_j^2/n}{n - 1}$$

Newman - Keuls Test

$$q = \frac{T_1 - T_2}{\sqrt{n\, MS_{error}}}$$

with degrees of freedom:
r = number of steps between ordered totals

f = df of MS error

Flesch Reading Ease Formula

$$R.E. = 1.6_s - 1.0_w - 31.5$$

s = average number of one syllable words per 100 words

w = average number of words per sentence.